

Homework problems:

2.1 (due Sep 21, 2006) There are 20 people at a party. Each person writes her/his name on a card (suppose that all the people in the room have different names). The cards are then mixed and each person is given back a card.

- a) What is the expected number of people that get their card back?
- b) What is the probability that exactly one person gets his/her card back? (Evaluate your answer to 3 digits of accuracy, your answer should be of the form $0.xyz$, where x, y, z are decimal digits.)

2.2 (due Sep 21, 2006) Two players Alice and Bob play the following game. They flip a coin until one of the following happens:

- 1) there are three consecutive heads (H,H,H), or
- 2) there is a tail followed by two heads (T,H,H).

In case 1) Alice wins and Bob has to pay her \$6. In case 2) Bob wins and Alice has to pay him \$1. Who has the advantage in this game? What is Alice's expected payoff?

2.3 (due Sep 21, 2006) We are given k sorted lists A_1, \dots, A_k . The total number of elements in the lists is n . We would like to merge the lists into one sorted list B (the number of elements in B will be n). Give an algorithm which solves this problem in time $O(n \log k)$.

2.4 (due Sep 21, 2006) We are given two sorted arrays A and B , each containing n elements. Assume that the arrays do not contain duplicates, and the elements in A are different from elements in B . We would like to compute the median of $A \cup B$. For example if $A = [1, 2, 3, 4, 5]$ and $B = [6, 7, 8, 9, 10]$ then the median is 5; if $A = [1, 3, 5, 7, 9]$ and $B = [2, 4, 6, 8, 10]$ then the median is again 5. Give an $O(\log n)$ algorithm for this problem.

Bonus problem:

2.5 (due Sep 21, 2006) The algorithm from Exercise 2.3 can be used to construct the following sorting algorithm:

- split A into k parts,
- sort each of the parts recursively,
- merge the sorted lists.

What is the name of the algorithm when $k = 2$? What is the name of the algorithm when $k = n$? The recurrence for the algorithm is

$$T(n) = kT(n/k) + \Theta(n \log k). \tag{1}$$

Solve the recurrence (1) for any k .