

Homework problems:

6.1 (due Oct 19, 2006) Let $G = (V, E)$ be a digraph given in the adjacency-list representation (i. e., for each vertex $v \in V$ we have a (linked) list of out-neighbors of v).

1. Write pseudocode for a procedure which outputs an adjacency-list representation of the reverse digraph (i. e., G with each edge reversed). The procedure should run in time $O(|V| + |E|)$.
2. Write pseudocode for a procedure which outputs the adjacency-list representation of G in which the out-neighbors of each vertex are listed in the increasing order. The procedure should run in time $O(|V| + |E|)$.
3. Write pseudocode for a procedure which checks if G is undirected (i. e., the reverse of every $e \in E$ is also in E). The procedure should run in time $O(|V| + |E|)$.

6.2 (due Oct 19, 2006) Let N be an integer. The company W produces weights of weights $1, \dots, N$. To place an order the customer has to send a list of integers a_1, \dots, a_N . The company W will then send the customer a_i weights of weight i for each $i \in \{1, \dots, N\}$. A package sent through UPS can weigh at most N . The company now faces the following problem: how to pack the order into the smallest number of packages (so that no package weighs more than N)?

Their new employee (a graduate of U of R) suggested the following greedy algorithm: Take an empty package and repeatedly add the heaviest item which fits. When no item fits then seal the package, get a new empty package, and repeat.

1. Show (prove) that the greedy algorithm works for $N = 6$.
2. Show (find a counterexample) that the greedy algorithm does not work for $N = 10$.

6.3 (due Oct 19, 2006) Write a dynamic programming algorithm which for a given number n finds the smallest number of squares which sum to n (for example for $n = 7$ we need 4 squares ($7 = 2^2 + 1^2 + 1^2 + 1^2$), whereas for $n = 13$ we only need 2 squares ($13 = 3^2 + 2^2$)). Implement your algorithm and find all numbers from $\{1, 2, \dots, 100\}$ which need 4 squares. Use “The On-Line Encyclopedia of Integer Sequences” to find a formula for the numbers which need 4 squares.

6.4 (due Oct 19, 2006) We are given a sequence of n positive numbers a_1, \dots, a_n . Give an algorithm which finds the increasing subsequence of a_1, \dots, a_n with the maximal sum. (For example on input 1, 101, 2, 3, 100, 4, 5 your algorithm should output 1, 2, 3, 100.)

Bonus problem:

6.5 (due Oct 19, 2006)

Determine for which values of N does the greedy algorithm in Problem 6.2 work.