1. Given a nondeterministic transition function \( \text{RESULT}(a) \) (or \( \text{RESULT}(s, a) \)) and a utility function \( U(s) \):

   (a) Give an expression for the expected utility of performing action \( a \) given evidence \( e \).

   (b) State the principle of maximum expected utility formally and in words.
2. Define the expected monetary value (EMV) of a gamble or lottery \([p_1, v_1; \ldots; p_n, v_n]\), where the gamble pays value \(v_i\) with probability \(p_i\).

3. Is the expected monetary value of a gamble equal to its expected utility? Why or why not?
4. Tickets to a lottery cost $1. There are two possible prizes: a $10 payoff with probability 1/50, and a $1,000,000 payoff with probability 1/2,000,000.

(a) What is the expected monetary value of the lottery?

(b) When (if ever) is it rational to buy a ticket? Be formal. Assume $S_n$ is the state of having $n$ dollars, you currently have $k$ dollars (i.e., you are in state $S_k$), and $U(S_k) = 0$. State and justify any other assumptions you need to make about utilities.

(c) Studies show that lower income people buy more lottery tickets. How would you explain this using the current framework?
5. Suppose you are in the market for a new house. A house can be in good shape ($g$) or bad shape ($\neg g$). There are various tests and inspections that you could perform, each with an associated cost. The tests can indicate what shape the house is in.

Suppose you are considering buying house for $150,000. If it’s in good shape, you believe that its market value is really $200,000. If not, it will need $70,000 in repairs to get it into shape. You believe that it has 70% chance of being in good shape.

Suppose you only have time to perform at most one test, which costs $5000.

(a) Draw the decision network that represents this problem and justify your design.

(b) Calculate the expected net gain (in dollars) of buying the house, given no test is performed.
(c) Suppose you have some knowledge about the probability that the test will accurately reflect the shape of the house:

\[
P(\text{pass} \mid g) = 0.8
\]
\[
P(\text{pass} \mid \neg g) = 0.35
\]

That is, a house in good shape will pass the test 80% of the time (and fail it 20% of the time—a false negative), while a house in bad shape will nonetheless pass the test 35% of the time (a false positive). Compute the conditional probability for the shape of the house given whether the test is passed (that is, \(P(G \mid \text{Pass})\)).

(d) Calculate the optimal decisions for the cases of a pass and of a fail, and their expected utilities.
(e) What does this say about the test? What does it say about the optimal plan for buying the house?