CSC242: Intro to AI

Lecture 14
Project 2: Wumpus Agent
Project 2 Goals
Project 2 Goals

Experience writing an agent that operates in a partially observable world
Project 2 Goals

- Experience writing an agent that operates in a partially observable world
- Explore to acquire new knowledge
- Combine observations and background knowledge to decide what to do
- Hybrid wumpus agent: AIMA Figure 7.20
Project 2 Goals

- Experience writing an agent that operates in a partially observable world
- Experience the pros and cons of using logic as a knowledge representation
Project 2 Goals

- Experience writing an agent that operates in a partially observable world
- Experience the pros and cons of using logic as a knowledge representation
- Factored representation, intuitive semantics
- Expressivity vs. computability tradeoffs
Project 2 Goals

- Experience writing an agent that operates in a partially observable world
- Experience the pros and cons of using logic as a knowledge representation
- Experience writing and using a client-server program that exchanges information
  - Asynchronous operation
  - External vs. internal representations
Project 2 Goals

- Experience writing an agent that operates in a partially observable world
- Experience the pros and cons of using logic as a knowledge representation
- Experience writing and using a client-server program that exchanges information
  - Asynchronous operation
  - External vs. internal representations
Project 2 Goals

- Experience writing an agent that operates in a partially observable world
- Experience the pros and cons of using logic as a knowledge representation
- Experience writing and using a client-server program that exchanges information
Project 2 Deliverables

- Part 1: Agent (client) that plays against the server: 40%
- Part 2: Use propositional logic representation of world: 40%
- Part 3: Use FOL representation of world: 20%
Project 2 Deliverables

- Part 1: Agent (client) that plays against the server: 50%
- Part 2: Apply logical (rule-based) techniques to the problems the agent has to solve: 50%

- What problem are you solving?
- What techniques are you using?
- How well do they work (pros and cons)?
Logical and/or Rule-Based Techniques

- Complete reasoning over a full logical model of the wumpus world not feasible (for us)
- I’ve given you lots of code for representing, reading, and manipulating logical constructs
- Idea: Forward- and/or backward-chaining over definite (or Horn) knowledge bases
- Other rule-based approaches: c.f. Wikipedia on “Production Systems” (JESS, Drools, ...)

Project 2
Due Sun Mar 25 11:59PM

● Part 1: Agent (client) that plays against the server: 50%

● Part 2: Apply logical (rule-based) techniques to the problems the agent has to solve: 50%

● What problem are you solving?

● What techniques are you using?

● How well do they work (pros and cons)?
Probabilistic Reasoning
Probabilistic Reasoning
Hunt The Wumpus
\[
P[1][1] = false;
\]
\[
P[1][1] = false; \quad B[1][1] = false;
\]
<table>
<thead>
<tr>
<th></th>
<th>1,1</th>
<th>2,1</th>
<th>3,1</th>
<th>4,1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,1</td>
<td>OK</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1,2</td>
<td>OK</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1,3</td>
<td>OK</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1,4</td>
<td>OK</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>1,4</th>
<th>2,4</th>
<th>3,4</th>
<th>4,4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1,2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1,3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1,4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- **A**: Agent
- **B**: Breeze
- **G**: Glitter, Gold
- **OK**: Safe square
- **P**: Pit
- **S**: Stench
- **V**: Visited
- **W**: Wumpus

\[
P[1][1] = \text{false}; \quad B[1][1] = \text{false};
\]
\[
P[2][1] = \text{false};
\]
\[
P[1][2] = \text{false};
\]
<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1,4</td>
<td>2,4</td>
<td>3,4</td>
<td>4,4</td>
</tr>
<tr>
<td>1,3</td>
<td>2,3</td>
<td>3,3</td>
<td>4,3</td>
</tr>
<tr>
<td>1,2</td>
<td>2,2</td>
<td>3,2</td>
<td>4,2</td>
</tr>
<tr>
<td>1,1</td>
<td>2,1</td>
<td></td>
<td>3,1</td>
</tr>
</tbody>
</table>

A = Agent
B = Breeze
G = Glitter, Gold
OK = Safe square
P = Pit
S = Stench
V = Visited
W = Wumpus

P[1][1] = false;   B[1][1] = false;
P[2][1] = false;
P[1][2] = false;
<table>
<thead>
<tr>
<th></th>
<th>1,4</th>
<th>2,4</th>
<th>3,4</th>
<th>4,4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1,2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1,1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Notes:**
- **A** = Agent
- **B** = Breeze
- **G** = Glitter, Gold
- **OK** = Safe square
- **P** = Pit
- **S** = Stench
- **V** = Visited
- **W** = Wumpus

P[1][1] = false; B[1][1] = false;
P[2][1] = false; B[2][1] = true;
P[1][2] = false;
<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>G</th>
<th>P</th>
<th>S</th>
<th>W</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agent</td>
<td>Breeze</td>
<td>Glitter, Gold</td>
<td>Pit</td>
<td>Stench</td>
<td>Wumpus</td>
</tr>
</tbody>
</table>

| OK | Safe square |

| V | Visited |

| (a) | (b) |

```
P[1][1] = false; B[1][1] = false;
P[2][1] = false; B[2][1] = true;
P[1][2] = false; |
```
<table>
<thead>
<tr>
<th></th>
<th>1,1</th>
<th>1,2</th>
<th>1,3</th>
<th>1,4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>V</td>
<td>A</td>
<td>OK</td>
<td>OK</td>
</tr>
<tr>
<td>2,1</td>
<td>B</td>
<td>V</td>
<td>OK</td>
<td>OK</td>
</tr>
<tr>
<td>3,1</td>
<td>P?</td>
<td>V</td>
<td>OK</td>
<td>OK</td>
</tr>
<tr>
<td>4,1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2,2</td>
<td>P?</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3,2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4,2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

P[1][1] = false;  B[1][1] = false;
P[2][1] = false;  B[2][1] = true;
P[1][2] = false;
<table>
<thead>
<tr>
<th></th>
<th>1,1</th>
<th>1,2</th>
<th>1,3</th>
<th>1,4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,1</td>
<td>V</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>OK</td>
<td>B</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2,1</td>
<td></td>
<td></td>
<td></td>
<td>B</td>
</tr>
<tr>
<td></td>
<td></td>
<td>V</td>
<td>V</td>
<td></td>
</tr>
<tr>
<td>1,2</td>
<td></td>
<td></td>
<td>P?</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1,3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2,3</td>
<td></td>
</tr>
<tr>
<td>2,3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1,4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4,4</td>
</tr>
</tbody>
</table>

- **A** = Agent
- **B** = Breeze
- **G** = Glitter, Gold
- **OK** = Safe square
- **P** = Pit
- **S** = Stench
- **V** = Visited
- **W** = Wumpus

P[1][1] = false; B[1][1] = false;
P[2][1] = false; B[2][1] = true;
P[1][2] = false; B[1][2] = true;
<table>
<thead>
<tr>
<th></th>
<th>1,4</th>
<th>2,4</th>
<th>3,4</th>
<th>4,4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,3</td>
<td>P?</td>
<td>2,3</td>
<td>3,3</td>
<td>4,3</td>
</tr>
<tr>
<td>1,2</td>
<td>A</td>
<td>2,2</td>
<td>P?</td>
<td>4,2</td>
</tr>
<tr>
<td>1,1</td>
<td>B</td>
<td>2,1</td>
<td>B</td>
<td>P?</td>
</tr>
<tr>
<td></td>
<td>V</td>
<td>V</td>
<td>V</td>
<td>V</td>
</tr>
</tbody>
</table>

- **A** = Agent
- **B** = Breeze
- **G** = Glitter, Gold
- **OK** = Safe square
- **P** = Pit
- **S** = Stench
- **V** = Visited
- **W** = Wumpus

P[1][1] = false;   B[1][1] = false;
P[2][1] = false;   B[2][1] = true;
P[1][2] = false;   B[1][2] = true;
• A pit causes a breeze in all neighboring squares.
• Each square other than [1,1] contains a pit with probability 0.2.
Diagnostic rule: Toothache $\Rightarrow$ Cavity
Diagnostic rule: $\overrightarrow{Toothache} \Rightarrow \overrightarrow{Cavity}$

$Toothache \Rightarrow Cavity \lor GumProblem \lor Abcess \lor ...$
Diagnostic rule:  \( \text{Toothache} \implies \text{Cavity} \)

\[ \text{Toothache} \implies \text{Cavity} \lor \text{GumProblem} \lor \text{Abcess} \lor \ldots \]

Qualifications

Causal rule:  \( \text{Cavity} \implies \text{Toothache} \)
Approach

- Representing uncertain information
- Reasoning with uncertain information
Uncertainty
Sources of Uncertainty

Partial Observability

Nondeterminism
Sources of Uncertainty

Partial Observability and Nondeterminism
Rational Action

• In the face of uncertainty, an agent still has to act.

• The right thing to do (the rational decision) depends on the relative importance of the various goals and the likelihood that they will be achieved (and to what degree).
Probability
WARNING!

MATH AHEAD
Possible Worlds

Hungry=true, Cranky=false

Hungry=false, Cranky=true

Hungry=true, Cranky=true

Hungry=false, Cranky=false
Hungry $\lor$ Cranky

- Hungry=true, Cranky=false
- Hungry=false, Cranky=true
- Hungry=true, Cranky=true
- Hungry=false, Cranky=false
Hungry $\Rightarrow$ Cranky

- Hungry = true, Cranky = false (Possible World)
- Hungry = false, Cranky = true (Possible World)
- Hungry = true, Cranky = true
- Hungry = false, Cranky = false
Possible Worlds

• Logical assertions rule out possible worlds
Possible Worlds

• Logical assertions rule out possible worlds
• Probabilistic assertions talk about how probable (likely) the possible worlds are
Sample Space

- In probability theory, the set of all possible worlds is called the sample space:
  \[ \Omega = \{ \omega_i \} \]

- Possible worlds \( \omega_i \) are:
  - Mutually exclusive
  - Exhaustive
\[ \Omega = \{ \omega_i \} = \{ (1,1), (1,2), (2,1), (3,1), \ldots \} \]
Probability Model

• Assigns a numerical probability $P(\omega)$ to each possible world*, such that:

\[ 0 \leq P(\omega) \leq 1 \]

\[ \sum_{\omega \in \Omega} P(\omega) = 1 \]

*Discrete, countable set of worlds
Where Do Probabilities Come From?

- Assigns a numerical probability $P(\omega)$ to each possible world*, such that:

\[ 0 \leq P(\omega) \leq 1 \]
\[ \sum_{\omega \in \Omega} P(\omega) = 1 \]

*Discrete, countable set of worlds
Degrees of Belief

- The degree to which an agent believes a possible world is the actual world
  - From 0: certainly not the case (i.e., false)
  - To 1: certainly is the case (i.e., true)
- Could come from statistical data, general principles, combination of evidence, ...
Probability Model

• Assigns a numerical probability $P(\omega)$ to each possible world*, such that:

\[ 0 \leq P(\omega) \leq 1 \]
\[ \sum_{\omega \in \Omega} P(\omega) = 1 \]

*Discrete, countable set of worlds
\[ P(\omega_i) = \frac{1}{36} \text{ for all } \omega_i \in \Omega \]
$P(\omega_i) = \frac{2}{36}$ if $\omega_i \in \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$

$P(\omega_i) = \frac{2}{90}$ otherwise
Propositions (Events)

• A proposition (event) corresponds to the set of possible worlds in which the proposition holds

\[ P(\phi) = \sum_{\omega \in \phi} P(\omega) \]
\[ P(Total = 11) = P((5, 6)) + P((6, 5)) = \frac{1}{36} + \frac{1}{36} = \frac{1}{18} \]
\[ P(Doubles) = \frac{1}{4} \]

\[ P((1, 1)) + P((2, 2)) + P((3, 3)) + P((4, 4)) + P(5, 5) + P((6, 6)) = \frac{1}{4} \]
Probability (so far)

- Sample space (possible worlds)
- Probability model (degrees of belief in worlds)
- Propositions (subsets of worlds in which proposition holds)
Programming Language for Uncertainty

Propositions + Probabilities $\Rightarrow$ Probability Statements
Unconditional (Prior) Probabilities

- Degrees of belief in propositions in the absence of any other information

\[ P(Doubles) = \frac{1}{4} \]
\[ P(\text{cavity}) = 0.2 \]
Conditional (Posterior) Probability

• Degree of belief in a proposition given some information (evidence)

\[ P(SnakeEyes \mid Die_1 = 1) = \frac{1}{4} \]
\[ P(cavity \mid toothache) = 0.6 \]

• Whenever evidence is true and we have no further information, conclude probability of proposition
Conditional Probability

\[ P(a \mid b) = \frac{P(a \land b)}{P(b)} \text{ when } P(b) > 0 \]
20 equally likely possible worlds
12 possible worlds where \( b \) is true \( P(b) = \frac{12}{20} \)
\( a \) is true in 6 of those 12 \( P(a \mid b) = \frac{6}{12} = \frac{1}{2} \)
20 equally likely possible worlds
12 possible worlds where \( b \) is true \( P(b) = 12/20 \)
6 worlds where \( a \land b \) is true: \( P(a \land b) = 6/20 \)

\[
P(a \mid b) = \frac{P(a \land b)}{P(b)} = \frac{6/20}{12/20} = 1/2
\]
Conditional Probability

\[ P(a \mid b) = \frac{P(a \land b)}{P(b)} \quad \text{when } P(b) > 0 \]
Product Rule

\[ P(a \land b) = P(a \mid b)P(b) \]
Probability

Probability: Degree of belief in possible world

\[ 0 \leq P(\omega) \leq 1 \]

\[ \sum_{\omega \in \Omega} P(\omega) = 1 \]

Probability of a proposition: Conditional probability:

\[ P(\phi) = \sum_{\omega \in \phi} P(\omega) \]

\[ P(a \mid b) = \frac{P(a \land b)}{P(b)} \text{ when } P(b) > 0 \]

\[ P(a \land b) = P(a \mid b)P(b) \]
Incomplete Information

Factored Representation

Variables & Values (domains)
Random Variables

• Take on values from a domain

• Boolean random variables have domain
  \{ true, false \}

• Domains can be finite or infinite
  • Discrete, infinite: integers
  • Continuous, infinite: reals
Random Variables

$Die_1 : \{1, 2, 3, 4, 5, 6\}$

$Total : \{2, \ldots, 12\}$

$Doubles : \{true, false\}$

$Weather : \{sunny, rain, cloudy, snow\}$
Atomic Propositions

- Restriction on possible values of a random variable (a.k.a. constraint)
- Including statement that a random variable takes on a particular value (i.e., domain restricted to a single value)
Atomic Propositions

Boolean random variables:

\[ \text{Doubles} = \text{true} \rightarrow \text{doubles} \]
\[ \text{Doubles} = \text{false} \rightarrow \neg\text{doubles} \]

Symbolic (unambiguous) value:

\[ \text{Weather} = \text{sunny} \rightarrow \text{sunny} \]

Ordered domains:

\[ 50 \leq \text{Weight} < 100 \]
\[ \text{NumberOfAtomsInUniverse} \geq 10^{70} \]
Connectives

Same connectives as propositional logic:
\[ \land, \lor, \rightarrow, \leftrightarrow \]

\[ P(\text{cavity} \mid \neg \text{toothache} \land \text{teen}) = 0.1 \]

\[ P(\text{Cavity} = \text{true} \mid \text{Toothache} = \text{false} \land \text{Age} = \text{teen}) = 0.1 \]
The Language of Probability Assertions

- Random variables (and domains)
- Atomic propositions:
  - \( \text{Var} = \text{value} \) (or \(<, \leq, >, \geq\))
- Connectives
Probabilities and Propositions

\[ P(\phi) = \sum_{\omega \in \phi} P(\omega) \]
The Language of Probability Assertions

Weather : \{sunny, rain, cloudy, snow\}

\[ P(Weather = \text{sunny}) = 0.6 \]
\[ P(Weather = \text{rain}) = 0.1 \]
\[ P(Weather = \text{cloudy}) = 0.29 \]
\[ P(Weather = \text{snow}) = 0.01 \]
Probability Distributions

- Describe the probabilities of all the possible values of a random variable

\[ P(Weather = \text{sunny}) = 0.6 \]
\[ P(Weather = \text{rain}) = 0.1 \]
\[ P(Weather = \text{cloudy}) = 0.29 \]
\[ P(Weather = \text{snow}) = 0.01 \]

\[ \mathbf{P}(\text{Weather}) = \langle 0.6, 0.1, 0.29, 0.01 \rangle \]
Joint Distributions

- Distributions over multiple variables
- Describe probabilities of all combinations of the values of the variables
Joint Distributions

\[ P(\text{Weather, Cavity}) \]

<table>
<thead>
<tr>
<th>Weather</th>
<th>Cavity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>true</td>
</tr>
<tr>
<td>sunny</td>
<td></td>
</tr>
<tr>
<td>rain</td>
<td></td>
</tr>
<tr>
<td>cloudy</td>
<td></td>
</tr>
<tr>
<td>snow</td>
<td></td>
</tr>
</tbody>
</table>
Joint Distributions

\[ P(\text{sunny, Cavity}) \]

<table>
<thead>
<tr>
<th>Weather</th>
<th>sunny</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cavity</td>
<td></td>
</tr>
<tr>
<td>true</td>
<td></td>
</tr>
<tr>
<td>false</td>
<td></td>
</tr>
</tbody>
</table>
Joint Distributions

\[ P(sunny, cavity) \]

\[ P(sunny, cavity) = P(sunny, cavity) = P(sunny \land cavity) \]

<table>
<thead>
<tr>
<th>Weather</th>
<th>sunny</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Cavity</td>
<td>true</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Conditional Distribution

$P(X \mid Y)$ gives the values of $P(X = x_i \mid Y = y_i)$ for each $i, j$ pair.
Continuous Distributions

- Probability density functions (PDFs)

\[ P(x) = \lim_{dx \to 0} \frac{P(x \leq X \leq x + dx)}{dx} \]
Full Joint Probability Distribution

• Joint probability distribution over all the random variables
• Probabilities for every possible combination of values assigned to random variables
• Probabilities for every possible world
### Full Joint Probability Distribution

\[ P(\text{Cavity}, \text{Toothache}, \text{Weather}) \]

<table>
<thead>
<tr>
<th></th>
<th>toothache</th>
<th>¬toothache</th>
</tr>
</thead>
<tbody>
<tr>
<td>cavity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>¬cavity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>sunny</td>
<td></td>
<td></td>
</tr>
<tr>
<td>rain</td>
<td></td>
<td></td>
</tr>
<tr>
<td>cloudy</td>
<td></td>
<td></td>
</tr>
<tr>
<td>snow</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Programming Language for Knowledge 3.0

- Probabilities
- Language of probability assertions
- Distributions, conditional distributions, joint distributions, full joint distributions
Probabilistic Inference
Probabilistic Inference

• Computing posterior probabilities for propositions given prior probabilities and observed evidence
\[ P(Cavity, \text{Toothache}, \text{Catch}) \]

<table>
<thead>
<tr>
<th></th>
<th>toothache</th>
<th>(\neg\text{toothache} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>catch</td>
<td>0.108</td>
<td>0.012</td>
</tr>
<tr>
<td>(\neg\text{catch} )</td>
<td>0.072</td>
<td>0.008</td>
</tr>
<tr>
<td>cavity</td>
<td>0.016</td>
<td>0.064</td>
</tr>
<tr>
<td>(\neg\text{cavity} )</td>
<td>0.144</td>
<td>0.576</td>
</tr>
</tbody>
</table>
Probability of a Proposition

\[ P(\text{cavity} \lor \text{toothache}) = 0.28 \]

<table>
<thead>
<tr>
<th></th>
<th>toothache</th>
<th>(\neg)toothache</th>
</tr>
</thead>
<tbody>
<tr>
<td>catch</td>
<td>0.108</td>
<td>0.012</td>
</tr>
<tr>
<td>(\neg)catch</td>
<td>0.072</td>
<td>0.008</td>
</tr>
<tr>
<td>cavity</td>
<td>0.016</td>
<td>0.064</td>
</tr>
<tr>
<td>(\neg)cavity</td>
<td>0.144</td>
<td>0.576</td>
</tr>
</tbody>
</table>
Probability of a Proposition

\[ P(\text{cavity} \land \text{toothache}) = 0.12 \]

<table>
<thead>
<tr>
<th></th>
<th>toothache</th>
<th>(\neg)toothache</th>
</tr>
</thead>
<tbody>
<tr>
<td>catch</td>
<td>0.108</td>
<td>0.012</td>
</tr>
<tr>
<td>(\neg)catch</td>
<td>0.072</td>
<td>0.008</td>
</tr>
<tr>
<td>cavity</td>
<td>0.016</td>
<td>0.064</td>
</tr>
<tr>
<td>(\neg)cavity</td>
<td>0.144</td>
<td>0.576</td>
</tr>
</tbody>
</table>
Conditional Probability

\[ P(\text{cavity} \mid \text{toothache}) = \frac{P(\text{cavity} \land \text{toothache})}{P(\text{toothache})} \]

<table>
<thead>
<tr>
<th>cavity</th>
<th>toothache</th>
<th>catch</th>
<th>¬catch</th>
<th>catch</th>
<th>¬catch</th>
</tr>
</thead>
<tbody>
<tr>
<td>cavity</td>
<td>0.108</td>
<td>0.012</td>
<td>0.072</td>
<td>0.008</td>
<td></td>
</tr>
<tr>
<td>¬cavity</td>
<td>0.016</td>
<td>0.064</td>
<td>0.144</td>
<td>0.576</td>
<td></td>
</tr>
</tbody>
</table>
Conditional Probability

\[ P(\text{cavity} \mid \text{toothache}) = \frac{P(\text{cavity} \land \text{toothache})}{P(\text{toothache})} = 0.12 \]

<table>
<thead>
<tr>
<th></th>
<th>toothache</th>
<th>(\neg\text{toothache} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-\text{cavity})</td>
<td>catch</td>
<td>0.016</td>
</tr>
<tr>
<td></td>
<td>(-\text{catch})</td>
<td>0.064</td>
</tr>
</tbody>
</table>
Conditional Probability

\[ P(\text{cavity} \mid \text{toothache}) = \frac{P(\text{cavity} \land \text{toothache})}{P(\text{toothache})} = \frac{0.12}{0.2} \]

<table>
<thead>
<tr>
<th></th>
<th>toothache</th>
<th>¬toothache</th>
</tr>
</thead>
<tbody>
<tr>
<td>catch</td>
<td>0.108</td>
<td>0.072</td>
</tr>
<tr>
<td>¬catch</td>
<td>0.012</td>
<td>0.008</td>
</tr>
<tr>
<td>¬cavity</td>
<td>0.016</td>
<td>0.144</td>
</tr>
<tr>
<td>catch</td>
<td>0.064</td>
<td>0.576</td>
</tr>
<tr>
<td>¬catch</td>
<td>0.144</td>
<td>0.576</td>
</tr>
</tbody>
</table>
\[
P(cavity \mid toothache) = \frac{P(cavity \land toothache)}{P(toothache)} = 0.6
\]

\[
\begin{array}{|c|c|c|c|c|}
\hline
 & toothache & \neg toothache \\
\hline
catch & 0.108 & 0.012 & 0.072 & 0.008 \\
\hline
\neg cavity & 0.016 & 0.064 & 0.144 & 0.576 \\
\hline
\end{array}
\]
Conditional Probability

\[ P(\text{cavity} \mid \text{toothache}) = \frac{P(\text{cavity} \land \text{toothache})}{P(\text{toothache})} = 0.6 \]

\[ P(\neg\text{cavity} \mid \text{toothache}) = \frac{P(\neg\text{cavity} \land \text{toothache})}{P(\text{toothache})} \]
### Conditional Probability

\[
P(\neg\text{cavity} \mid \text{toothache}) = \frac{P(\neg\text{cavity} \land \text{toothache})}{P(\text{toothache})} = 0.4
\]

<table>
<thead>
<tr>
<th></th>
<th>toothache</th>
<th>\neg\text{toothache}</th>
</tr>
</thead>
<tbody>
<tr>
<td>catch</td>
<td>0.108</td>
<td>0.012</td>
</tr>
<tr>
<td>\neg\text{catch}</td>
<td>0.016</td>
<td>0.064</td>
</tr>
<tr>
<td>cavity</td>
<td>0.072</td>
<td>0.008</td>
</tr>
<tr>
<td>\neg\text{cavity}</td>
<td>0.144</td>
<td>0.576</td>
</tr>
</tbody>
</table>
Conditional Probability

\[ P(\text{cavity} \mid \text{toothache}) = \frac{P(\text{cavity} \land \text{toothache})}{P(\text{toothache})} = 0.6 \]

\[ P(\neg\text{cavity} \mid \text{toothache}) = \frac{P(\neg\text{cavity} \land \text{toothache})}{P(\text{toothache})} = 0.4 \]

\[ P(\text{Cavity} \mid \text{toothache}) = \langle 0.6, 0.4 \rangle \]
Probabilistic Inference

\[ P(\text{cavity} \mid \text{toothache}) = \frac{P(\text{cavity} \land \text{toothache})}{P(\text{toothache})} = 0.6 \]

\[ P(\neg \text{cavity} \mid \text{toothache}) = \frac{P(\neg \text{cavity} \land \text{toothache})}{P(\text{toothache})} = 0.4 \]
Normalization

\[ P(\text{cavity} \mid \text{toothache}) = \alpha P(\text{cavity} \land \text{toothache}) = \alpha 0.12 \]

\[ P(\neg\text{cavity} \mid \text{toothache}) = \alpha P(\neg\text{cavity} \land \text{toothache}) = \alpha 0.08 \]
Normalization

\[ P(\text{cavity} \mid \text{toothache}) = \alpha \] \[ P(\text{cavity} \land \text{toothache}) = \alpha 0.12 \]

\[ P(\neg \text{cavity} \mid \text{toothache}) = \alpha \] \[ P(\neg \text{cavity} \land \text{toothache}) = \alpha 0.08 \]

\[ P(\text{Cavity} \mid \text{toothache}) = \alpha \langle 0.12, 0.08 \rangle \]
Normalization

\[ P(\text{cavity} \mid \text{toothache}) = \alpha P(\text{cavity} \land \text{toothache}) = \alpha 0.12 \]

\[ P(\neg \text{cavity} \mid \text{toothache}) = \alpha P(\neg \text{cavity} \land \text{toothache}) = \alpha 0.08 \]

\[ P(\text{Cavity} \mid \text{toothache}) = \frac{1}{0.12 + 0.08} \langle 0.12, 0.08 \rangle \]
Normalization

\[ P(cavity \mid toothache) = \alpha P(cavity \land toothache) = \alpha 0.12 \]

\[ P(\neg cavity \mid toothache) = \alpha P(\neg cavity \land toothache) = \alpha 0.08 \]

\[ P(Cavity \mid toothache) = \langle 0.6, 0.4 \rangle \]

We didn’t know \( P(toothache) \)!
Inference
(Single Variable)

\[ P(X \mid e) = \alpha \ P(X, e) = \alpha \sum_y P(X, e, y) \]

Query variable \( X : \text{Domain}(X) = \{x_1, \ldots, x_m\} \)

Evidence variables \( E : \{E_1, \ldots, E_k\} \)

Observations \( e : \{e_1, \ldots, e_k\} \) s.t. \( E_i = e_i \)

Unobserved variables \( Y : \{Y_1, \ldots, Y_l\} \)

\( \text{Domain}(Y_i) = \{y_{i,1}, \ldots, y_{i,n_i}\} \)
Inference (Single Variable)

\[ P(X \mid e) = \alpha \, P(X, e) = \alpha \sum_y P(X, e, y) \]

For each possible value \( x_i \) for \( X \)

For each possible combination of values \( y \) for \( Y \)

Add \( P(x_i, e, y) \)

Result: vector \( P(X \mid e) = \langle P(x_i \mid e) \rangle = \langle P(X = x_i \mid e) \rangle \)
Inference
(Single Variable)

\[ P(X \mid e) = \alpha \sum_y P(X, e, y) \]

For each possible value \( x_i \) for \( X \)

For each possible combination of values \( y \) for \( Y \)

Add \( P(x_i, e, y) \)

\[ P(X = x_i, E_1 = e_1, \ldots, E_k = e_k, \ldots, Y_1 = y_{1,i_1}, \ldots, Y_l = y_{l,i_l}) \]

Result: vector \( P(X \mid e) = \langle P(x_i \mid e) \rangle = \langle P(X = x_i \mid e) \rangle \)
### Inference (Single Variable)

\[
P(X | e) = \alpha \ P(X, e) = \alpha \sum_{y} P(X, e, y)
\]

```c
int m;    // Number of values in domain of query variable X
int k;    // Number of evidence variables E
int[k] e; // e[i]: index of i’th evidence value in domain of Ei
int l;    // Number of unobserved variables Y
int[l] n; // n[i]: number of values in domain of Yi
double[l][n[l]] D; // D[i][j]: j’th value of domain for Yi

for i from 1 to m
    PXe[i] = 0
    for i1 from 1 to n[1]
        for i2 from 1 to n[2]
            ...
            for il from 1 to n[l]
                PXe[i] += JPDF[i][e[1]]...[e[k]][D[1][i1]]...[D[l][il]]
    return PXe // vector of length m
```
Inference
(Single Variable)

\[
P(X \mid e) = \alpha P(X, e) = \alpha \sum_y P(X, e, y)
\]

Time Complexity
\[O(m^n)\]

Space Complexity
\[O(m^n)\]
Summary

- Uncertainty arises from partial observability and/or non-determinism
- Probability model (for us) assigns a degree of belief to possible worlds; satisfies axioms of probability theory
- Probabilities:
  - Unconditional (prior)
  - Conditional (posterior, given evidence)
Summary

- Factored representation: Random variables, $Var=value$ (atomic props), connectives
- Distributions: probabilities for all the values of a variable
- Set of variables: Joint distribution
- Inference: Computing posterior (conditional) probabilities for propositions given observed evidence
For Next Time:
AIMA Ch 14-14.5