Quiz

Stop Time: 2:15
ULW

- First draft due Mar 1
- 8-10 pages minimum
First-Order Logic
First-Order Logic
Propositional Logic

- Syntax & Semantics
- Truth tables
- Model checking
- Inference
- Inference rules
- Resolution
Project 2: Logical Sudoku

- Formulate sudoku using logic
- Write a program that inputs a sudoku and outputs its logical representation
- Solve the puzzle using an inference engine
  - Which you don’t have to write...
- Write a program that outputs the solution of the sudoku nicely
- Due: Mon 4 Mar 23:00
Propositional Logic

- Syntax & Semantics
- Truth tables
- Model checking
- Inference
- Inference rules
- Resolution
• Rooms adjacent to pits will have breezes

• Socrates is a person
  All people are mortal

• Anybody’s grandmother is either their mother’s or their father’s mother
Logic 2.0

• Define a language based on propositional logic that will allow us to say all these things

• Define entailment ("follows from")

• Find inference rules that will allow us to compute the consequences of our knowledge (entailments)
“Socrates is a person”

P socrates_is_a_person
“Socrates is a person”

P \ socrates_is_a_person

“George is a person”

Q \ george_is_a_person

“Fido is a dog”

R \ fido_is_a_dog

“Dogbert is a dog”

S \ dogbert_is_a_dog

“Dogbert is an executive”

T \ dogbert_is_an_executive
Object-Oriented

• Objects: Socrates, George, Fido, Dogbert
• Classes: Person, Dog, Executive, ...

Socrates = new Person()  
George = new Person()  
Fido = new Dog()  
Dogbert = new (Dog() and Executive())?
Constants

• Symbols denoting objects in the world

• Socrates, George, Fido, Dogbert, ...

denote  |diˈnōt|
verb [ trans. ]
be a sign of; indicate: *this mark denotes purity and quality.*
• (often be denoted) stand as a name or symbol for: *the level of output per firm, denoted by X.*
Predicates

• Denote relationships between objects

• \textit{Predicate}(arg\ _1, \ arg\_2, \ldots, \ arg\_n)

• \textbf{Man}(Socrates), \textbf{Dog}(Fido)

• \textbf{Adjacent}(George, Fido)

• \textbf{Between}(Cheese, Burger, Bun)

• \textbf{Dog}(Dogbert) and \textbf{Executive}(Dogbert)
“Anybody’s grandmother is either their mother’s or their father’s mother”
Functions

• Names for objects built out of other objects

• \textit{function}(arg1, arg2, \ldots, argn)

• \textit{boss}(George)

• \textit{mother}(Socrates)

• \textit{room}(1,2)
• “George’s father is Dogbert’s boss”
father(George) = boss(Dogbert)

• “Socrates’ mother is a person”
Person(mother(Socrates))

• “Room 1,2 is adjacent to room 1,1”
Adjacent(room(1,2), room(1,1))
• Constants that denote objects in the world

• Function symbols that denote a mapping from a tuple of objects to another object

• Predicate symbols that denote relations (sets of tuples of objects)

• Term = constant or function+terms
Connectives

- Negation (not, $\neg$)
- Conjunction (and, $\land$)
- Disjunction (or, $\lor$)
- Implication (if-then, $\Rightarrow$)
- Biconditional (if and only if, $\leftrightarrow$)
• Person(George) ∧ Dog(Fido)
• Cat(Dogbert) ∧ Dog(Dogbert)
• ¬ Mouse(Dogbert)
• Adjacent(room(2,1), room(1,1)) ∧ Adjacent(room(2,1), room(2,2)) ∧ ...
• Dog(Dogbert) ⇒ Mammal(Dogbert)
• Dog(Dogbert) ⇒ Barks(Dogbert)
• Dog(Dogbert) ⇒ Mammal(Dogbert)
• Dog(Dogbert) ⇒ Barks(Dogbert)
• Dog(Scooby) ⇒ Mammal(Scooby)
• Dog(Scooby) ⇒ Barks(Scooby)
• “All dogs are mammals”

• “All dogs bark”

• “All people are mortal”

• “All rooms adjacent to pits are breezy”
Set<Dog> dogs;
for (x : dogs) {
    setIsMammal(x, true);
}

my @dogs;
foreach my $x (@dogs) {
    $isMammal{$x} = 1;
}
Variables

• Denote objects, but we don’t know which

• Are terms: can appear wherever a constant or function expression can

• $\text{Dog}(x)$

• $\text{Happy}(\text{father}(y))$

• $\text{Adjacent}(\text{room}(x,y), \text{room}(+(x,1),y))$
Quantifiers

- If $\alpha$ is a sentence, then so are
  - $\forall x \alpha$
  - $\exists x \alpha$
Universal quantification

• $\forall x \, \alpha$ is true if $\alpha$ is true for all possible values of $x$

• $\forall x \, \text{Dog}(x) \Rightarrow \text{Mammal}(x)$

• $\forall y \, \text{Dog}(y) \land \text{Mammal}(y)$
Existential quantification

• $\exists x \alpha$ is true if $\alpha$ is true for some possible values of $x$

• $\exists x \text{Manages}(x, \text{Dogbert})$

• $\exists r \text{Room}(r) \land \text{In(Wumpus, r)}$
First-Order Logic

• Constants and variables that denote objects in the world
• Function symbols that denote a mapping from a tuple of objects to another object
• Predicate symbols that denote relations (sets of tuples of objects)
• Connectives
• Quantifiers
A Programming Language for Knowledge

• Syntax:
  • What counts as a well-formed statement, formula, sentence, or program

• Semantics:
  • What these statements, formulas, sentences, or programs mean
Semantics of First-Order Logic
Models for PL

- Assignment of truth values to propositional variables
## Models for PL

<table>
<thead>
<tr>
<th>$B_{1,1}$</th>
<th>$P_{1,2}$</th>
<th>$P_{2,1}$</th>
<th>$P_{1,2} \lor P_{2,1}$</th>
<th>$B_{1,1} \Leftrightarrow (P_{1,2} \lor P_{2,1})$</th>
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<tbody>
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First-Order Logic

- Constants and variables that denote objects in the world
- Function symbols that denote a mapping from a tuple of objects to another object
- Predicate symbols that denote relations (sets of tuples of objects)
- Connectives
- Variables and Quantifiers
FOL: Constants

• Refer to objects in the world
Tuesday, February 26, 13
Richard (1157-1199)

John (1166-1216)
Models for FOL: Domain

- Non-empty set of objects in the world
FOL: Constants

- Each constant refers to some element of the domain (some object in the world)
FOL: Predicates

- Refer to relations between objects
- Relations = tuples (ordered sets) of objects that are related
\[
\text{Person} = \{ \langle R \rangle, \langle J \rangle \}
\]
\[
\text{King} = \{ \langle J \rangle \}
\]
\[
\text{Crown} = \{ \langle C \rangle \}
\]
\[ \text{Brother} = \{ \langle R, J \rangle, \langle J, R \rangle \} \quad \text{OnHead} = \{ \langle C, J \rangle \} \]
Models for FOL: Relations

- Sets of tuples of objects
  - Unary: one object
  - Binary: pair of objects
  - Ternary: triple of objects
  - n-ary: n-tuple of objects
FOL: Functions

• Refer to objects in terms of other objects

• Functions = Total mapping from tuples of objects to objects
\( \text{LeftLegOf} = \{ \langle R \rangle \rightarrow LLR, \langle J \rangle \rightarrow LLJ \} \)
Models for FOL: Functions

- Total mappings from tuples of objects to objects
  - Unary: one object
  - Binary: pair of objects
  - Ternary: triple of objects
  - \( n \)-ary: \( n \)-tuple of objects
Models for FOL

• Domain: Non-empty set of objects in the world
• Relations: Sets of tuples of objects
• Functions: Total mappings from tuples of objects to objects
• Models are possible worlds

• We want to be able to compute whether a sentence is true in a model

• From that we can define entailment

• And then we can do inference (“follows from”)

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Interpretation

- Mapping from elements a sentence to elements of a model
- Specifies exactly which objects, relations, and functions are referred to by the constant, predicate, and function symbols
Interpretation

- $R$ refers to Richard the Lionheart
- $J$ refers to evil King John
- $\text{Brother}$ refers to the brotherhood relation (that is, the set of tuples we saw before); similarly $\text{OnHead}$, $\text{Person}$, $\text{King}$, and $\text{Crown}$
- $\text{LeftLeg}$ refers to the “left leg of” function (that is, the mapping we saw before)
Models for FOL

• Domain: Non-empty set of objects in the world

• Interpretation that maps:
  • Constants to objects
  • Predicate symbols to relations between objects
  • Function symbols to total functions on objects
Atomic Sentences

• Predicate symbol + list of terms (arguments)
  • $\text{Brother}(R, J)$
  • $\text{Married(}\text{Father}(\text{Richard}), \text{Mother}(\text{John}))$

• True in a model if the relation referred to by the predicate symbol holds among the objects referred to by the arguments
Interpretation

- $R$ refers to Richard the Lionheart
- $J$ refers to evil King John
- $Brother$ refers to the brotherhood relation (that is, the set of tuples we saw before); similarly $OnHead, Person, King$, and $Crown$
- $LeftLeg$ refers to the “left leg of” function (that is, the mapping we saw before)
\( \text{Brother} = \{ \langle R, J \rangle, \langle J, R \rangle \} \) \hspace{1cm} \text{OnHead} = \{ \langle C, J \rangle \} \\

\text{Brother}(R, J) \quad \checkmark
\[ \text{Brother} = \{ \langle R, J \rangle, \langle J, R \rangle \} \quad \text{OnHead} = \{ \langle C, J \rangle \} \]

\( \text{Brother}(C,J) \) \( \times \)
Atomic Sentences

• Predicate symbol + list of terms (arguments)
  
  • \textit{Brother}(R, J)
  
  • \textit{Married}(\textit{Father}(Richard), \textit{Mother}(John))
  
• True in a model if the relation referred to by the predicate symbol holds among the objects referred to by the arguments
Complex Sentences

• Connectives combine sentences
  • \( \neg \text{Brother}(\text{LeftLeg}(R), J) \)
  • \( \text{Brother}(R, J) \land \text{Brother}(J, R) \)
  • \( \neg \text{King}(R) \Rightarrow \text{King}(J) \)

• Same semantics as propositional logic!

• True in a model if truth values of arguments satisfies truth table for connective
First-Order Logic

• Constants and variables that denote objects in the world
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• Connectives
First-Order Logic

- Constants and variables that denote objects in the world
- Function symbols that denote a mapping from a tuple of objects to another object
- Predicate symbols that denote relations (sets of tuples of objects)
- Connectives
- Variables and Quantifiers
• “All dogs are mammals”
• “All dogs bark”
• “All people are mortal”
• “All rooms adjacent to pits are breezy”
• “Some room contains the wumpus”
Quantifiers

• If $\alpha$ is a sentence, then so are

• $\forall x \alpha$ (universal quantifier)

• $\exists x \alpha$ (existential quantifier)
Universal Quantifier

• \( \forall x \, P \) says that \( P \) is true for every object \( x \)

• \( \forall x \, P \) is true in a model if \( P \) is true in all possible extended interpretations of the interpretation in the model

• Each extended interpretation maps \( x \) to an object in the domain of the model
• \( \forall x \; \text{King}(x) \Rightarrow \text{Person}(x) \)

• Intended interpretation:
  • Domain: Richard, John, crown, Richard’s left leg, John’s left leg

• Extended interpretations:
  • \( x \rightarrow \text{Richard} \)
  • \( x \rightarrow \text{John} \)
  • \( x \rightarrow \text{the crown} \)
  • \( x \rightarrow \text{Richard’s left leg} \)
  • \( x \rightarrow \text{John’s left leg} \)
• $\forall x \text{ King}(x) \Rightarrow \text{ Person}(x)$

• **Extended interpretations:**
  • Richard is a king $\Rightarrow$ Richard is a person
  • John is a king $\Rightarrow$ John is a person
  • The crown is a king $\Rightarrow$ The crown is a person
  • Richard’s left leg is a king $\Rightarrow$ Richard’s left leg is a person
  • Richard’s left leg is a king $\Rightarrow$ Richard’s left leg is a person
Universal Quantifier

• $\forall x \, P$ says that $P$ is true for every object $x$

• $\forall x \, P$ is true in a model if $P$ is true in all possible extended interpretations of the interpretation in the model

• Each extended interpretation maps $x$ to an object in the domain of the model
Existential Quantifier

- $\exists x \ P$ says that $P$ is true for some object $x$

- $\exists x \ P$ is true in a model if $P$ is true in at least one possible extended interpretation of the interpretation in the model

- Each extended interpretation maps $x$ to an object in the domain of the model
• $\exists x \text{ Crown}(x) \land \text{ OnHead}(x, J)$

• Intended interpretation:
  • Domain: Richard, John, crown, Richard’s left leg, John’s left leg

• Extended interpretations:
  • $x \rightarrow$ Richard
  • $x \rightarrow$ John
  • $x \rightarrow$ the crown
  • $x \rightarrow$ Richard’s left leg
  • $x \rightarrow$ John’s left leg
\[ \exists x \text{ Crown}(x) \land \text{OnHead}(x,J) \]

- Extended interpretations:
  - Richard is a crown \land Richard is on John’s head
  - John is a crown \land John is on John’s head
  - The crown is a crown \land The crown is on John’s head
  - Richard’s left leg is a crown \land Richard’s left leg is on John’s head
  - Richard’s left leg is a crown \land Richard’s left leg is on John’s head
Existential Quantifier

- $\exists x \ P$ says that $P$ is true for some object $x$
- $\exists x \ P$ is true in a model if $P$ is true in at least one possible extended interpretation of the interpretation in the model
- Each extended interpretation maps $x$ to an object in the domain of the model
Nested Quantifiers

- “Brothers are siblings”
  - $\forall x \forall y \text{Brother}(x,y) \Rightarrow \text{Sibling}(x,y)$

- “Everybody loves somebody”
  - $\forall x \exists y \text{Loves}(x,y)$

- “Somebody is loved by everybody”
  - $\exists y \forall x \text{Loves}(x,y)$
Models for FOL

- Domain: Non-empty set of objects in the world
- Interpretation that maps:
  - Constants to objects
  - Predicate symbols to relations between objects
  - Function symbols to total functions on objects
Entailment

• $\alpha$ entails $\beta$ when:
  • $\beta$ is true in every world considered possible by $\alpha$
  • Every model of $\alpha$ is also a model of $\beta$
  • $\text{Models}(\alpha) \subseteq \text{Models}(\beta)$
All Possible Models

• # of objects from 1 to \( \infty \)
• Some constants refer to the same object
• Some objects are not referred to by any constant ("unnamed")
• Relations and functions defined over sets of subsets of objects
• Variables range over all possible objects in extended interpretations
137,506,194,466 models with \( \leq 6 \) objects!
Computing Entailment

• Number of models (probably) unbounded
  • And anyway hard to evaluate truth in a model
• Can’t do model checking
• Look for inference rules, do theorem proving
For Next Time
AIMA Ch. 9

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