

# PRACTICE PROBLEM SET #5

CSC 261/461 (Database Systems), Spring 2018,  
University of Rochester  
02/28/2018

## Problem 1

Refer: Section 15.1.2

Let  $F = \{A \rightarrow C, AC \rightarrow D, E \rightarrow AD, E \rightarrow H\}$ .

Let  $G = \{A \rightarrow CD, E \rightarrow AH\}$ .

Show that:

1. G covers F
2. F covers G
3. F and G are equivalent

### Solution:

1. Show: G covers F or ( $F \subseteq G^+$ )

let's check each FD in  $F$ :

- $A \rightarrow C$   
So, let's find  $A^+$  in  $G$ .  
 $A^+$  in  $G = \{ACD\}$  which includes  $C$ . So, continue;
- $AC \rightarrow D$   
Let's get  $AC^+$  in  $G$ ,  
 $AC^+$  in  $G = \{ACD\}$  which contains  $D$ . So, continue;
- $E \rightarrow AD, E \rightarrow H$   
Let's get  $E^+$  in  $G$ ,  
 $E^+$  in  $G = \{EACDH\}$  which contains  $AD$  and  $H$ . So, Done;

We found that every dependency in  $F$  can be inferred from  $G$ .  
So, we can say that  $G$  covers  $F$ .

2. Show: F covers G or ( $G \subseteq F^+$ )

let's check each FD in  $G$ :

- $A \rightarrow CD$   
So, let's find  $A^+$  in  $F$ .  
 $A^+$  in  $F = \{ACD\}$  which includes  $CD$ . So, continue;
- $E \rightarrow AH$   
Let's get  $E^+$  in  $F$ ,  
 $E^+$  in  $F = \{EACDH\}$  which contains  $AH$ . So, continue;

We found that every dependency in  $G$  can be inferred from  $F$ .  
So, we can say that  $F$  covers  $G$ .

3. As  $G$  covers  $F$  (from 1) and  $F$  covers  $G$  (from 2), so,  $F$  and  $G$  are equivalent ( $F \equiv G$ ).

## Problem 2

Refer: Section 15.1.3

Let the relation R be R(A,B,C,D,E) and the given set of FDs be

$F : \{A \rightarrow D, BC \rightarrow AD, C \rightarrow B, E \rightarrow A, E \rightarrow D\}$ .

Find the minimal cover of F

### Solution:

- Step 1: Convert all dependencies in canonical form (that is, they have only one attribute on the right hand side).

$\{A \rightarrow D, BC \rightarrow A, BC \rightarrow D, C \rightarrow B, E \rightarrow A, E \rightarrow D\}$

- Step 2: If left-hand-side contains multiple attributes, check if any of those can be removed. We have  $BC \rightarrow A$ , so we will check if  $B \rightarrow A$  or  $C \rightarrow A$  is sufficient. For this, we need to get  $B^+$  and  $C^+$ .  $B^+ = \{B\}$ , and  $C^+ = \{ABCD\}$ . As,  $B^+$  does not have A, so, C cannot be removed. But  $C^+$  contains A. So, We do not need B and  $C \rightarrow A$  is sufficient.

We get:

$\{A \rightarrow D, C \rightarrow A, BC \rightarrow D, C \rightarrow B, E \rightarrow A, E \rightarrow D\}$

Similarly, we can get rid of B from  $BC \rightarrow D$  as  $C^+$  contains D

We get:

$\{A \rightarrow D, C \rightarrow A, C \rightarrow D, C \rightarrow B, E \rightarrow A, E \rightarrow D\}$

- Step 3: This is the fun part. We will check if we can get rid of any of these dependencies completely. For that, we will iterate over each FD,  $X \rightarrow Y$ . Calculate  $X^+$  without considering  $X \rightarrow Y$ . If we find  $Y \subset X^+$ , we can infer  $X \rightarrow Y$  is redundant (because we found some other FD/FDs Z such that,  $X \rightarrow Z$  and  $Z \rightarrow Y$ ).

So, let's do it.

- $A \rightarrow D$ :

Calculate  $A^+$  without  $A \rightarrow D$ :

We get:  $A^+ = \{A\}$ .. So,  $A^+$  does not contain D. We cannot remove it.

- $C \rightarrow A$ :

Calculate  $C^+$  without  $C \rightarrow A$ :

We get:  $C^+ = \{CBD\}$ .. So,  $C^+$  does not contain A. We cannot remove it.

- $C \rightarrow D$ :

Calculate  $C^+$  without  $C \rightarrow D$ :

We get:  $C^+ = \{CAD\}$ .. So,  $C^+$  contains D. We can remove it.

Current cover:  $\{A \rightarrow D, C \rightarrow A, C \rightarrow B, E \rightarrow A, E \rightarrow D\}$

- $C \rightarrow B$ :

Calculate  $C^+$  without  $C \rightarrow B$ :

We get:  $C^+ = \{CAD\}$ .. So,  $C^+$  does not contain B. We cannot remove it.

- $E \rightarrow A$ :

Calculate  $E^+$  without  $E \rightarrow A$ :

We get:  $E^+ = \{ED\}$ .. So,  $E^+$  does not contain A. We cannot remove it.

- $E \rightarrow D$ :

Calculate  $E^+$  without  $E \rightarrow D$ :

We get:  $E^+ = \{EAD\}$ .. So,  $E^+$  contains D. We **can** remove it.

Current cover:  $\{A \rightarrow D, C \rightarrow A, C \rightarrow B, E \rightarrow A\}$

So, minimal cover is:

$\{A \rightarrow D, C \rightarrow A, C \rightarrow B, E \rightarrow A\}$

or

$\{A \rightarrow D, C \rightarrow AB, E \rightarrow A\}$

### Problem 3

Let the relation schema  $R(A,B,C,D)$  is given. For each of the following set of FDs do the following:

i) indicate all the BCNF violations. Do not forget to consider FD's that are not in the given set. However, it is not required to give violations that have more than one attribute on the right side.

ii) Decompose the relations, as necessary, into collections of relations that are in BCNF.

1. FDs  $AB \rightarrow C$ ,  $C \rightarrow D$ , and  $D \rightarrow A$
2. FDs  $B \rightarrow C$ ,  $B \rightarrow D$
3. FDs  $AB \rightarrow C$ ,  $BC \rightarrow D$ ,  $CD \rightarrow A$ ,  $AD \rightarrow B$

#### Solution:

1. There are 14 nontrivial dependencies, They are:  $C \rightarrow A$ ,  $C \rightarrow D$ ,  $D \rightarrow A$ ,  $AB \rightarrow D$ ,  $AB \rightarrow C$ ,  $AC \rightarrow D$ ,  $BC \rightarrow A$ ,  $BC \rightarrow D$ ,  $BD \rightarrow A$ ,  $BD \rightarrow C$ ,  $CD \rightarrow A$ ,  $ABC \rightarrow D$ ,  $ABD \rightarrow C$ , and  $BCD \rightarrow A$ .

There are three keys —  $AB$ ,  $BC$ , and  $BD$ . Thus, any dependency above that does not have one of these pairs on the left is a BCNF violation. These are:  $C \rightarrow A$ ,  $C \rightarrow D$ ,  $D \rightarrow A$ ,  $AC \rightarrow D$ , and  $CD \rightarrow A$ .

One choice is to decompose using the violation  $C \rightarrow D$ . Using the above FDs, we get  $ACD$  (Because of  $C \rightarrow D$  and  $C \rightarrow A$ ) and  $BC$  as decomposed relations.  $BC$  is surely in BCNF, since any two-attribute relation is. we discover that  $ACD$  is not in BCNF since  $C$  is its only key. We must further decompose  $ACD$  into  $AD$  and  $CD$ . Thus, the three relations of the decomposition are  $BC$ ,  $AD$ , and  $CD$ .

2. By computing the closures of all 15 nonempty subsets of  $ABCD$ , we can find all the nontrivial FDs. They are  $B \rightarrow C$ ,  $B \rightarrow D$ ,  $AB \rightarrow C$ ,  $AB \rightarrow D$ ,  $BC \rightarrow D$ ,  $BD \rightarrow C$ ,  $ABC \rightarrow D$  and  $ABD \rightarrow C$ . From the closures we can also deduce that the only key is  $AB$ . Thus, any dependency above that does not contain  $AB$  on the left is a BCNF violation. These are:  $B \rightarrow C$ ,  $B \rightarrow D$ ,  $BC \rightarrow D$  and  $BD \rightarrow C$ .

One choice is to decompose using the violation  $B \rightarrow C$ . Using the above FDs, we get  $BCD$  and  $AB$  as decomposed relations.  $AB$  is surely in BCNF, since any two-attribute relation is. we discover that  $BCD$  is in BCNF since  $B$  is its only key and the projected FDs all have  $B$  on the left side. Thus the two relations of the decomposition are  $AB$  and  $BCD$ .

3. There are 12 nontrivial dependencies, including the four given ones and the eight derived ones. They are  $AB \rightarrow C$ ,  $BC \rightarrow D$ ,  $CD \rightarrow A$ ,  $AD \rightarrow B$ ,  $AB \rightarrow D$ ,  $AD \rightarrow C$ ,  $BC \rightarrow A$ ,  $CD \rightarrow B$ ,  $ABC \rightarrow D$ ,  $ABD \rightarrow C$ ,  $ACD \rightarrow B$  and  $BCD \rightarrow A$ .

We also found out that the keys are AB, AD, BC, and CD. Thus, any dependency above that does not have one of these pairs on the left is a BCNF violation. However, all of the FDs contain a key on the left so there are no BCNF violations. No decomposition is necessary since all the FDs do not violate BCNF.