CSC 280 Quiz 2 Finite Automata (Make-Up)

Name: ________________________________________________________________

The purpose of this quiz is to demonstrate that you understand how to prove properties of regular languages by constructing or modifying finite automata.

Use the scratch paper at the end of this booklet to work out the details of each proof, and then copy the final proof neatly in space provided.

Turn in the quiz at the start of class on Tuesday 14 Feb. Your final grade for the Quiz 2 will be average of your score on the original quiz and on this quiz.
1. Let $\Sigma = \{0,1\}$ and let $D = \{ w \mid w$ contains an equal number of occurrences of the substrings 01 and 10 $\}$. Thus $101 \in D$ because 101 contains a single 01 and a single 10, but $1010 \notin D$ because 1010 contains two occurrences of 10 and only one of 01. Prove that $D$ is regular by drawing a finite automata (deterministic or non-deterministic) that recognizes it, and explain in English why it works.
2. For languages $A$ and $B$, let the **perfect shuffle** of $A$ and $B$ be the language

$\{w | w = a_1 b_1 \ldots a_k b_k \text{ where } a_1 \ldots a_k \in A \text{ and } b_1 \ldots b_k \in B\}$. Prove that $w$ is regular by constructing an automata (deterministic or non-deterministic) that recognizes it, based on automata for $A$ and for $B$. Be sure to specify the transition function precisely.
3. If \( L_1 \) and \( L_2 \) are languages, define \( L_1 - L_2 \) as the set of strings that are in \( L_1 \) but not in \( L_2 \). Prove that if \( L_1 \) and \( L_2 \) are regular, so is \( L_1 - L_2 \). Do not construct an automata, prove this using properties of regular languages.