

## Exponent Rules

In the mathematical preliminary reading, these are referred to as “Weiss 1.2.1”.

$$x^a x^b = x^{a+b}$$

$$\frac{x^a}{x^b} = x^{a-b}$$

$$(x^a)^b = x^{ab}$$

$$x^n + x^n = 2x^n \neq x^{2n}$$

$$2^n + 2^n = 2^{n+1}$$

## Logarithm Rules

In the mathematical preliminary reading, these are referred to as “Weiss 1.2.2”.

Def:

$x^a = b$  if and only if  $\log_x b = a$ .

$$\log_a b = \frac{\log_c b}{\log_c a}, c > 0$$

$$\log ab = \log a + \log b$$

$$\log a/b = \log a - \log b$$

$$\log a^b = b \log a$$

$$\log 1 = 0$$

$$\log 2 = 1$$

$$\log_{1K}(1024) = 10$$

$$\log_{1M}(1,048,576) = 20$$

## Series

Same as “Weiss 1.2.3”.

$$\begin{aligned}\sum_{i=1}^n f(n) &= n f(n) \\ \sum_{i=t}^n f(i) &= \sum_{i=1}^n f(i) - \sum_{i=1}^{t-1} f(i) \\ \sum_{i=0}^n 2^i &= 2^{n+1} - 1 \\ \sum_{i=0}^n a^i &= \frac{a^{n+1} - 1}{a - 1} \\ \sum_{i=0}^n a^i &\leq \frac{1}{a - 1}, \quad 0 < a < 1 \\ \sum_{i=1}^{\infty} \frac{1}{2^i} &= 1 \\ \sum_{i=1}^{\infty} \frac{i}{2^i} &= 2 \\ \sum_{i=1}^n i &= \frac{n(n+1)}{2} \approx n^2/2 \\ \sum_{i=1}^n i^2 &= \frac{n(n+1)(2n+1)}{6} \approx n^3/3 \\ \sum_{i=1}^n i^k &= \frac{n^{k+1}}{|k+1|}, \quad k \neq -1 \\ H_n &= \sum_{i=1}^n \frac{1}{i} \approx \log_e n\end{aligned}$$

## Modular Arithmetic

Same as “Weiss 1.2.4”.

Modular arithmetic deals with integers for which you have divided out some constant and are only dealing with the remainder. Modular arithmetic is good for calculating with cyclic arrangements, such as hours of the day. On the 24 hour clock, they go from 0 (midnight) to 23 (11pm) and start over. Thus 96 hours after 3 am is  $(3 + 96) \bmod 24$ , the remainder when 99 is divided by 24, or 3. Still 3 am, but 4 days later.

All integers congruent to some other number are part of an equivalence class induced by the mod operation....that is, all 3ams are equivalent, no matter what day they fall on.

Of course  $a$  is congruent to  $b$  modulo  $n$ , or  $a \equiv b \pmod{n}$  iff  $n$  divides  $a - b$  evenly. That is,  $a$  and  $b$  both have the same remainder when divided by  $n$ .

As with equality, if  $a \equiv b \pmod{n}$  then  $a + c \equiv b + c \pmod{n}$  and  $ad \equiv bd \pmod{n}$ .

## Pascal's Triangle

1  
1 1  
1 2 1  
1 3 3 1  
1 4 6 4 1  
1 5 10 10 5 1  
...

This gives the binomial coefficients  $\binom{n}{k}$ , with  $n$  running down the rows starting at 0 and  $k$  across the columns starting at 0. Note  $\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$ .