

ODE "Balls in the Air"



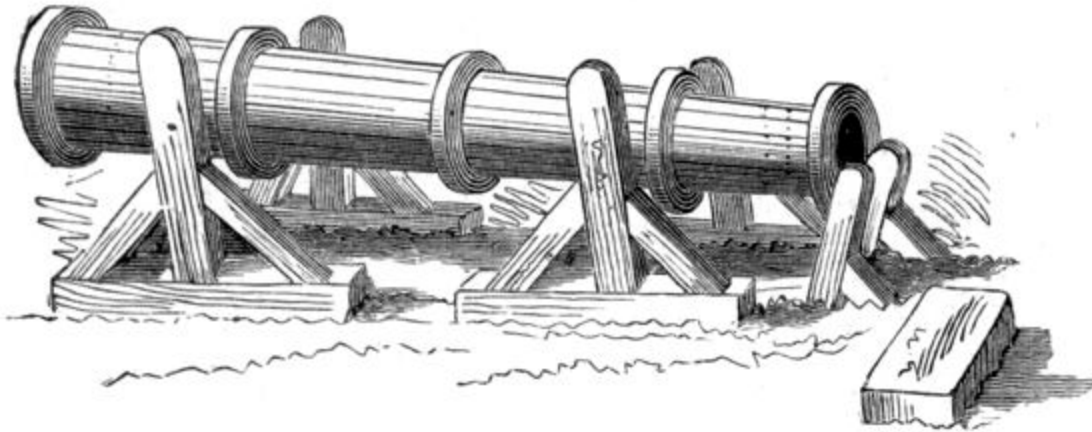
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Introduction

This lab write-up covers the “Balls in the Air” option of the ODE Matlab assignment. “Balls in the air, deals almost exclusively with examples that stem from The Battle of Crecy, a battle between French and English forces in the early 14th Century.



A representation of an English cannon that was used during the battle [1]

These cannons were made of wood and were often loaded with grapeshot or very large arrows, similar to those loaded in a ballista.

Constants:

For a majority of these problems, we will be using a series of constants unless stated otherwise. These are the following:

$V_i = 150 \text{ m/s}$ (Muzzle velocity of projectile)

$M = 5 \text{ kg}$ (Mass of projectile)

$g = 9.81 \text{ m/s}^2$ (acceleration of gravity)

$\rho = 1.23 \text{ kg/m}^3$ (density of medium (air))

$c = .5$ (drag coefficient of projectile)

Density of iron = 7860 kg/m^3

Deriving the diameter of the ball from mass and density:

$$\begin{aligned} \text{Volume} &= \text{Mass} / \text{Density} \\ \text{Volume} &= 5 / 7860 \\ \text{Volume} &= 0.0006361323155216285 \text{ m}^3 \\ \text{Volume} &= (4/3) \cdot \pi \cdot r^3 \quad (\text{solve for } r) \\ r &= 0.05335227582 \text{ m (radius of projectile)} \end{aligned}$$

Forming the ODE:

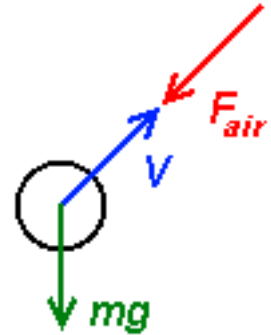
Drag Force:

$$F_D = \frac{1}{2} \rho u^2 C_D A$$

Source:[2]

Where:

- F_D is the force of drag on the object.
- ρ is the density medium (air for all cases)
- u is the relative velocity of the object
- C_D is the drag coefficient
- A is the reference area of the object



Velocity:

The conversion from unit vector motion into directional velocity:

$$v^2 = (x^2 + y^2 + z^2)$$

Where x, y and z are the component velocities

Reference Area:

The reference area of the object is calculated as:

$$A = \pi (d/2)^2$$

Where d is the diameter of the projectile

Using our constants from earlier we can calculate our new A constant is calculated as:

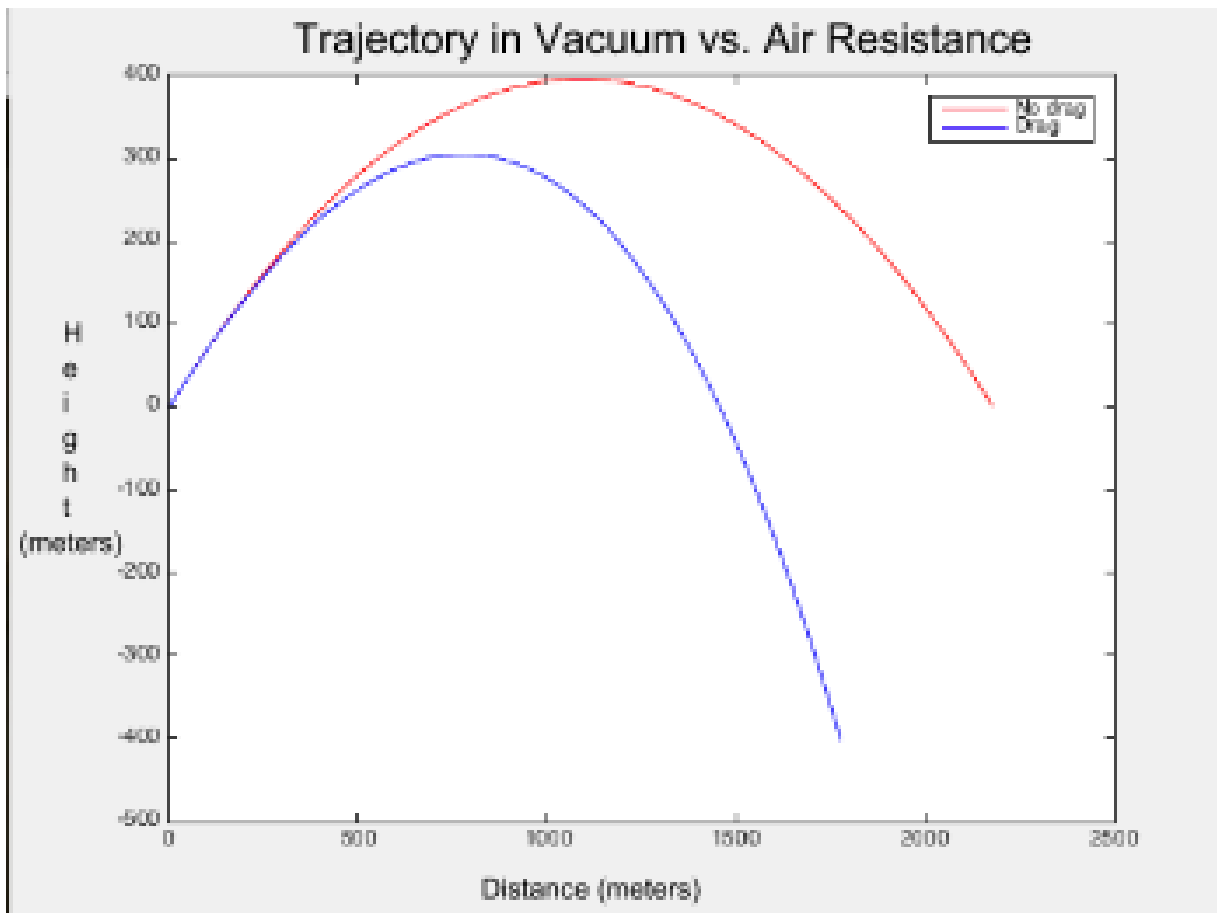
$$A = .0089$$

Using $F = M \cdot A$ we can form $A = F/M$. This along with the knowledge that the second derivative of distance is acceleration, we achieve the equations:

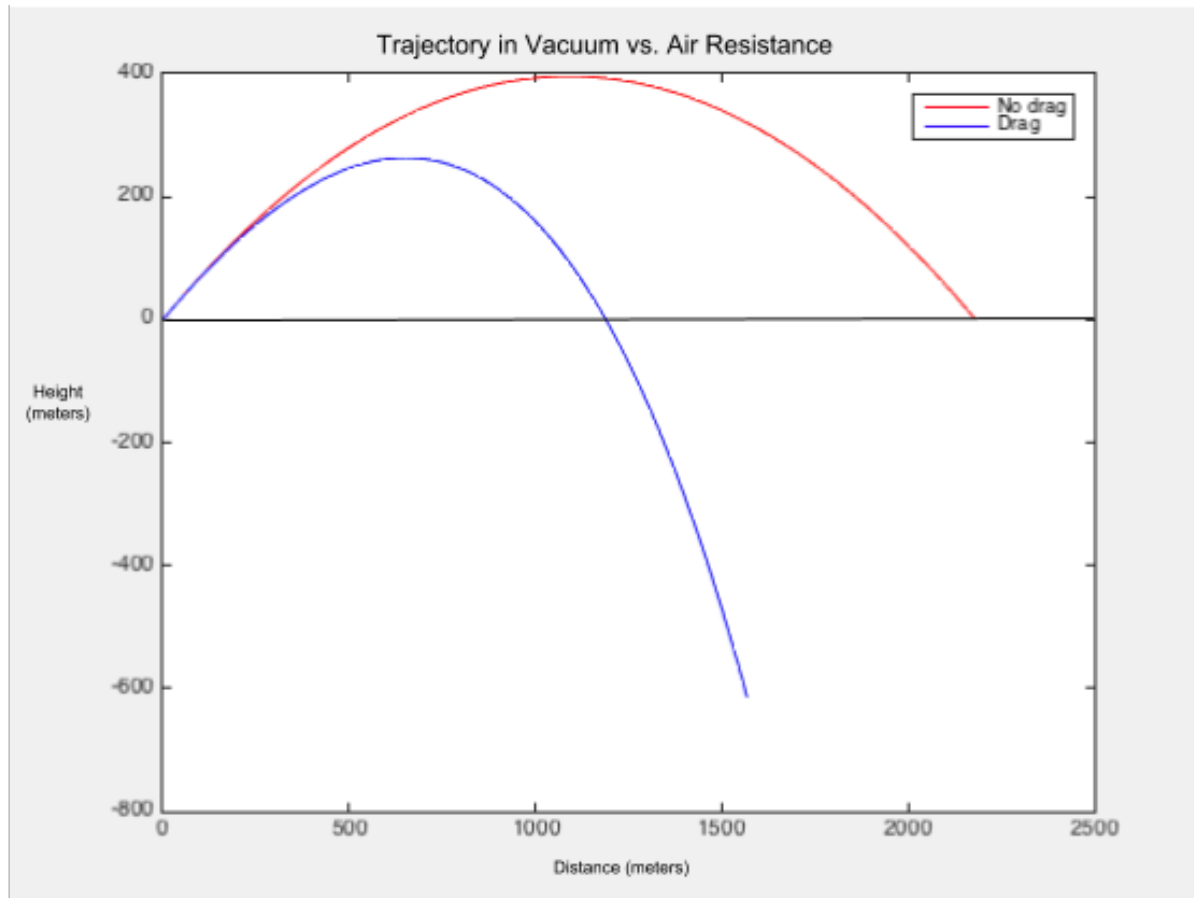
$$x'' = - (1/(2m)) (\rho c A s^2 \alpha) \text{ (drag alone)}$$
$$y'' = -g - (1/(2m)) (\rho c A s^2 \beta) \text{ (gravity plus drag)}$$

1. A Few Graphs

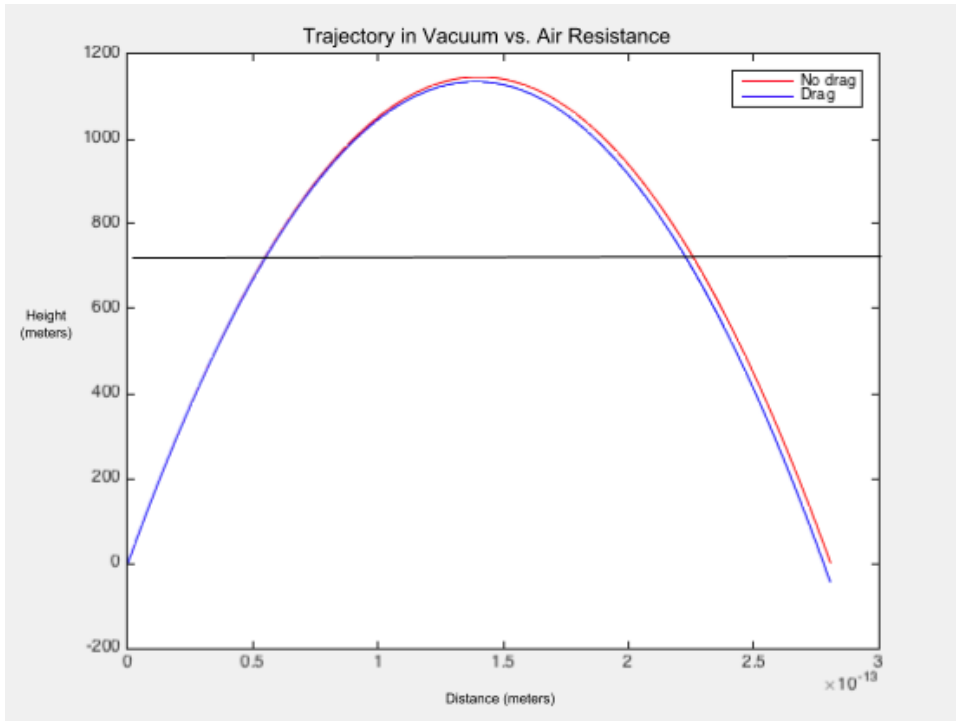
- A. The below graph shows a 45 degree launch and shows the comparisons between a drag coefficient of 0 and a drag coefficient of .5. Both use the same projectile.



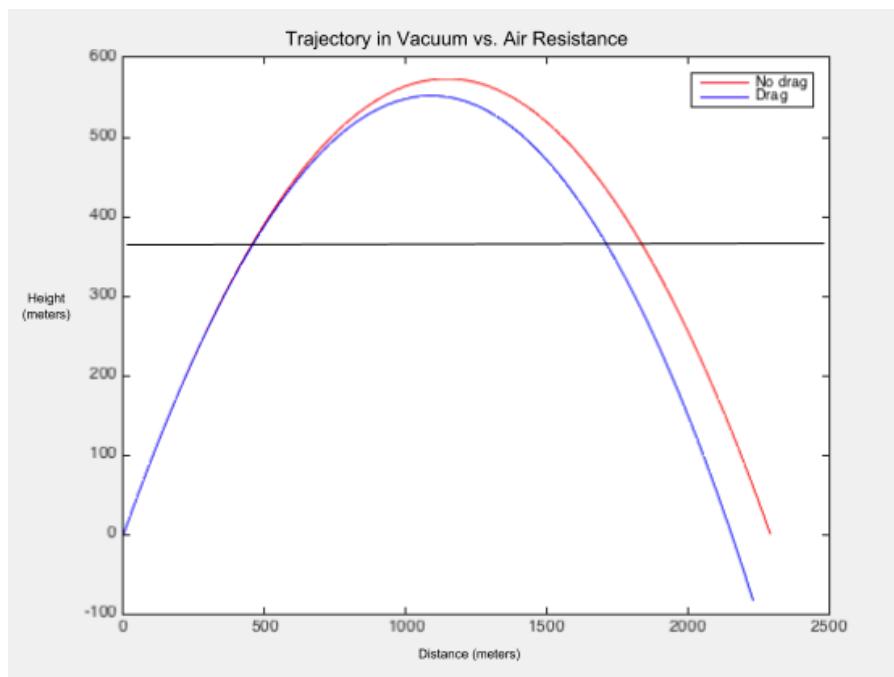
B. The coefficient was changed to .9 from .5 significantly decreasing the flight time. This fact is the key element of this assignment. A middle line was added to help determine exactly where the non vacuum projectile lands



C. When fired directly upward. Because of how the system plots the graphs the data it looks as though



D. Fired at 45 degrees with 80 mph winds in the positive direction.

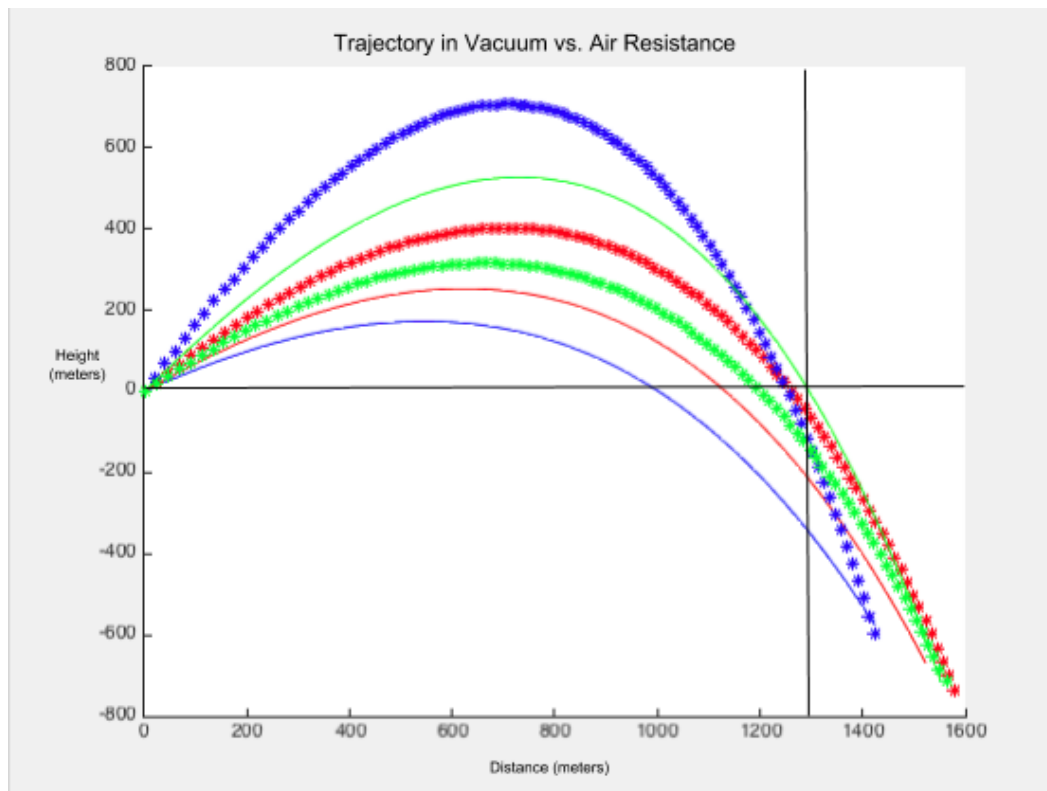


2. Angle and distance

In this question we are asked to determine the optimal angle for firing the cannon to achieve maximum range with a drag coefficient of .5 with the knowledge that “in a vacuum the optimal angle is 45 degrees. Given this information we can calculate trajectory through an ODE equation. After running each equation with respect to time. We can run them with respect to angle of elevation.

Results

As the coefficient of resistance is increases, the angle becomes more shallow. This is due to the hangtime of the projectile. At a steep angle, the projectile spends more time in the air, all the while being slowed down by the forces of friction. Thusly military cannons (arittlery) are often made to fire at angles between 27 - 36 degrees to attain the maximum range. This is only ever not true at high altitudes, where the air density is low enough that firing at incredibly high altitudes is preferable, in which the optimal angle is between 46-54 degrees.[3] It seems that at with a coefficient of .5 the maximum angle arrives at about 51.45 degrees.

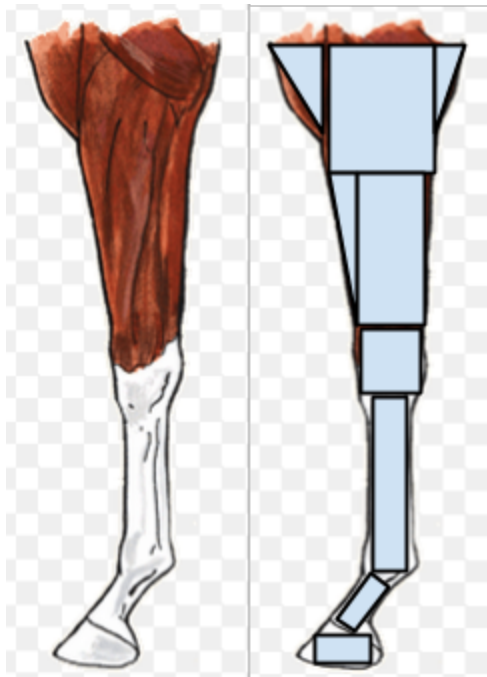


3. Psychological Warfare

Mass of the projectile:

During the battle of crecy there were a wide variety of troops and animals present, thusly the mass of the projectile could have been very prevalent. The battle took place on French soil, so i'm not putting it beyond the English troops to make use of French animals on nearby farmland. The main necessary decision is what kind of meat to use. Gathering animals would take time and effort, and would mean the abandoning the well defended english postion. This is something the heavily outnumbered English could not afford. According to the historic records of the battle, the first heavy engagement saw a small harassing force of cavalry maintaining distance between the French crossbowmen and the English positions. Over the course of this, the crossbowmen were the first to fall, followed by a disastrous cavalry attack against a caltrop laiden hill. So there were a large amount of two kinds of flesh available for the attack, A. Horse meat, B. Men-at-Arms.

For the greatest effect, the English would want the OLDEST, most rotten meat possible, so these first days casualties would be the ones picked. Unfortunately the distance between the English and the French positions would still bring the recovery troops far to close to the french encampments which would pick off the English artillerymen. The English would also already need to remove the horses from the hill defences to sally out (if necessary) as Prince Edward was fond of keeping options open [4]. So the cannons would use Horse flanks a weapon used throughout history.



Each of the horses legs weigh between 100kg-150kg. Depending on the horse, this can vary greatly. The French were fond of a breed (bred in Normandy) known as the Norman Cob[5] which are known to be very light. So for this case we will be using the low end of this range at 100kg.

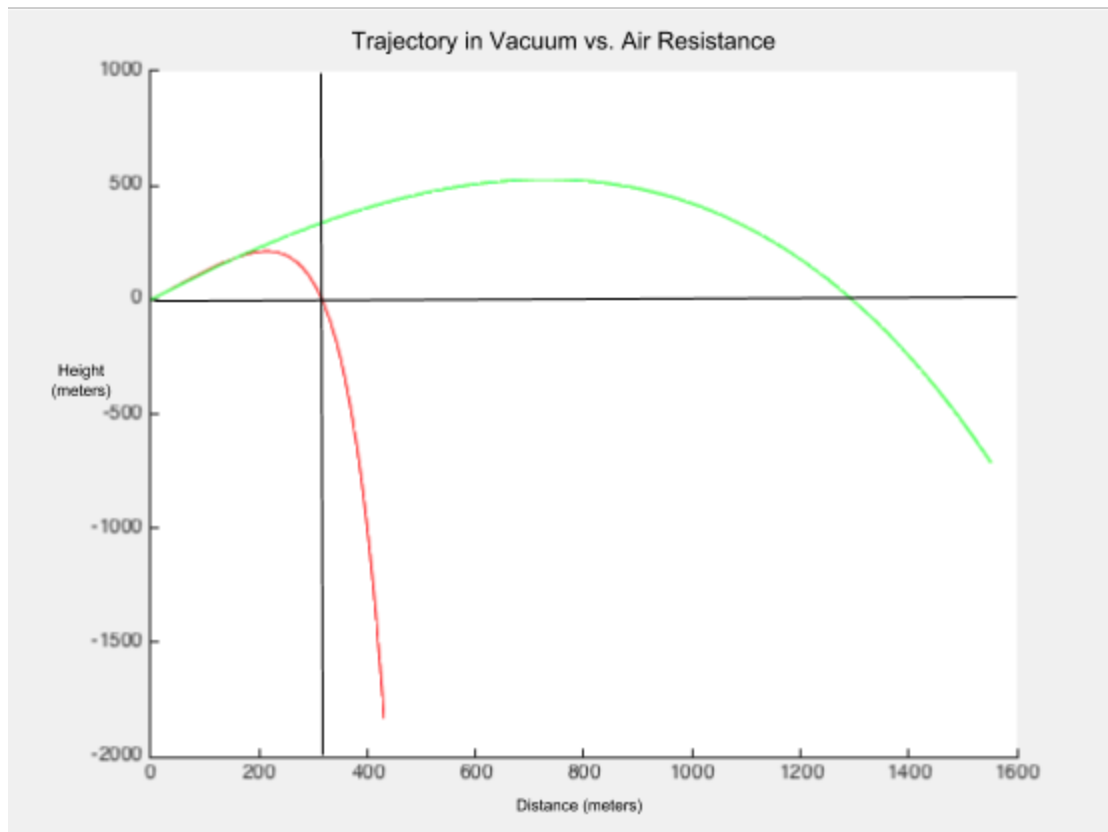
The area will also be an issue, which will take a significant effort to calculate. Modest estimates (using the height of the horses (measured in hands) comes out to about $4m^3$

This is important because horse density is significantly greater than that of a humans due to meat mass. Thus by applying it to

$$\begin{aligned} \text{Volume} &= (4/3) * \pi * r^3 \\ \text{Volume of Leg} &= \sim 4\text{m}^3 \\ \text{Mass of Leg} &= \sim 100\text{kg} \\ \text{Density of leg} &= \text{Mass/Volume} = 25\text{kg/m}^3 \\ \text{Mass of meatball} &= .353 \text{ kg} \end{aligned}$$

Results

Due to the significantly decreased mass, the effect of the drag force upon the projectile will be increased significantly. This would imply that the distance fired will be significantly shorter. With a 5kg shot, it was common for the shot to launch around 1200m with a 51.45 degree angle, as it was concluded to be the optimal angle for total distance. The shift from 45 degrees increased the maximum distance from 295.46m to 340.65m.



4. The Cocky Lieutenant (air density)

While the lieutenant out of line, his fears are for the most part well founded, with a few issues. Using the following equation you can take altitude and convert it to pressure

$$p = p_0 \left(1 - \frac{Lh}{T_0} \right)^{\frac{gM}{RL}}$$

Where:

- p = pressure at altitude
- p₀ = sea-level atmospheric pressure
- L = a pressure/altitude lapse rate
- T = is a sea-level temperature
- M = is molar mass of air
- R = Ideal gas constant

Afterward, we can apply it to the following which converts pressure to density:

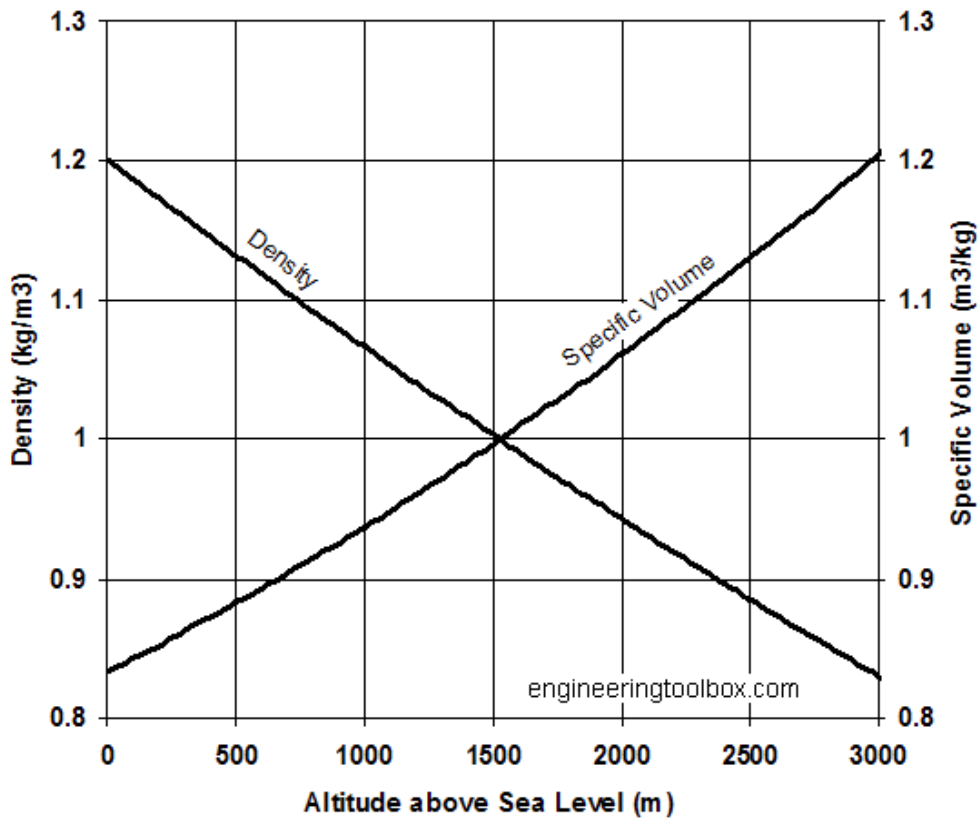
$$\rho = \frac{p}{R_{\text{specific}} T}$$

Where:

- p = the density of the air
- T = the temperature of the air
- R = Gas constant for dry air (Crecey was a dry battle)

These two equations give you a complete relationship between altitude and air density.

You can also gain a rough estimate through the following graph:



Unfortunately for our lieutenant even with our iron projectile fired directly upward in a vacuum, the maximum height change would be about 4500 meters. While this would mean practically no wind resistance at this point, there becomes no real measure when firing at any logical distance. Even in a vacuum, firing at a 90 degree angle, you achieve a maximum height change of 3430 feet. This decreases the air density by about .6. And while this could be important, the time spent at that altitude is fairly short.

5. Enter Z (Wind)

The most important note about understanding this part of the assignment is that the the winds effect is in a **separate axis to that of the motion**. This means that for all forces caused by wind (drag) the total velocity is at all times 80mph, which equates to 35.7632 m/s.[6]

So at all times the force acting upon the projectile in the x direction is:

$$F_d = (c A \rho v^2)/2$$
$$F_d = (.5 * .0089 * 1.23 * 35.7632^2)/2$$
$$F_d = 3.5003209683763195 \text{ N}$$

And thus the acceleration in the z direction is constant:

$$A_z = F/m$$
$$A_z = 3.5003209683763195/5$$
$$A_z = .7 \text{ m/s}^2$$

Due to the fact that the wind travels faster than the projectile in the direction of motion, there is no wind resistance. and thus the motion can be described by a simple kinematic equation:

$$\Delta Z = V_i * t + (\frac{1}{2} * a * t^2)$$

Where:

Z is the total Z displacement

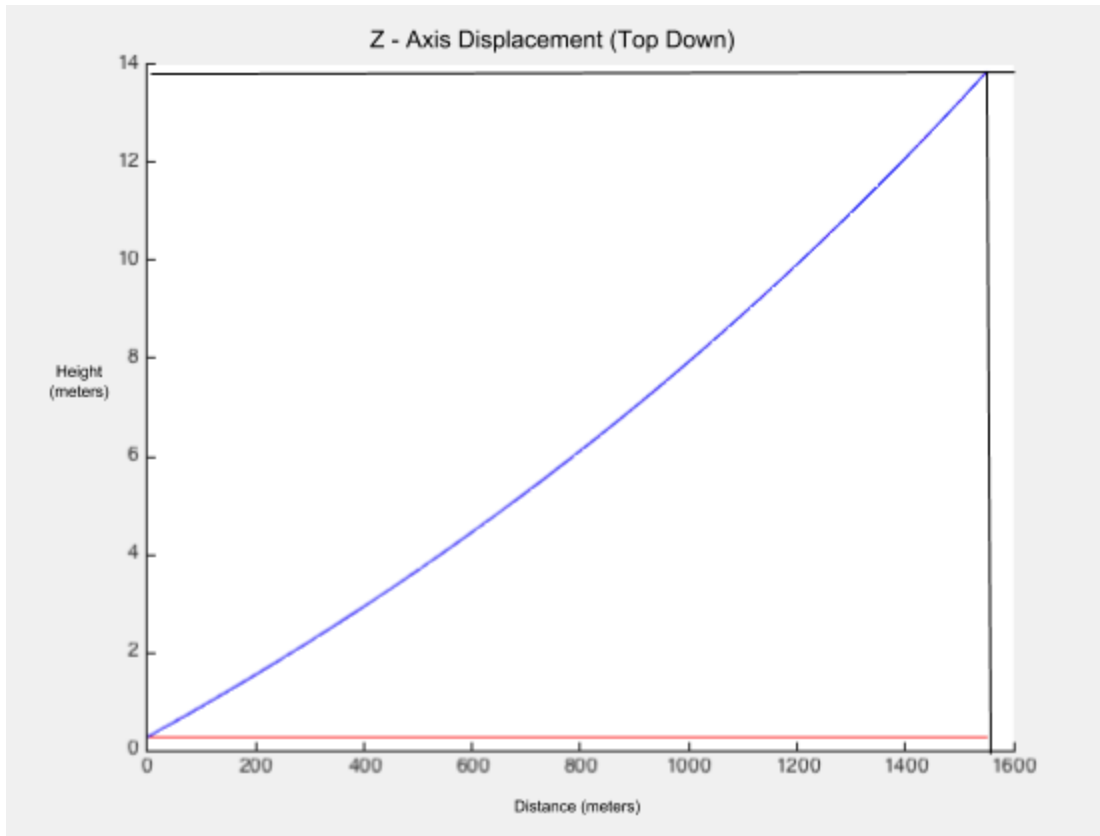
V_i is the initial z velocity (0)

a is the constant acceleration in the z direction (.7 m/s²)

t is the time of flight

By calculating an identical cannon shot with no z direction wind, we can determine this cases time of flight. and thus the total Z direction displacement off target.

In order to simulate this, the equations needed to be expanded to allow a third z equation to be applied, so as the plot was formed, I could create a trajectory of the system from a top down perspective. In this example, the final value of the z displacement in this example was 12.8828 meters. Here the red line represents no wind, and the z axis represents 80 mph wind, which when recalculated is 35.7632 m/s of wind.



Bibliography:

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