

3D Tic-Tac-Toe Assignment

MAKSIM ORLOVICH

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Contents

1	Introduction	1
2	Methology used.	1
3	Results.	2
3.1	Tournament Results	2
3.2	Summary of Statistical Data	2
4	Analysis	2
A	Statistical Information Logs	3
A.1	Minimax	3
A.1.1	Vs Spot	3
A.1.2	Vs $\alpha - \beta$	3
A.1.3	Vs $\alpha - \beta$ ordered	3
A.1.4	At $\alpha - \beta$	4
A.1.5	At $\alpha - \beta$	4
A.1.6	At $\alpha - \beta$ ordered	4
A.2	$\alpha - \beta$	5
A.2.1	Vs Spot	5
A.2.2	Vs Minimax	5
A.2.3	Vs $\alpha - \beta$ ordered	6
A.2.4	At Spot	6
A.2.5	At minimax	6
A.2.6	At $\alpha - \beta$ ordered	7
A.3	$\alpha - \beta$ ordered	7
A.3.1	Vs Spot	7
A.3.2	Vs Minimax	7
A.3.3	Vs $\alpha - \beta$	8
A.3.4	At Spot	8
A.3.5	At Minimax	8
A.3.6	At $\alpha - \beta$	9

1 Introduction

Three algorithms were implemented to play three dimensional tic-tac-toe – basic minimax search, basic alpha-beta search, and alpha-beta search with an ordering heuristic. In order to compare their performance, a tournament between them and the reference random-move making test program (aka 'Spot') was run, with matches monitored for signs of bugs, and extensive log information collected.

All 3 algorithms used the same threat-space evaluator, with the policy that if 2 players are in the same threat line, the threat is nullified. Since the evaluator is non-monotonic (after a win for one side, the score may return to zero), inner nodes were evaluated as well, so detect winning. The ordering function used in the ordered version of $\alpha - \beta$ search evaluated nodes 1 level below using the evaluator, and sorting them in decreasing order of score for the player whose nodes are considered.

2 Methodology used.

First, each algorithm was tested in human play, to test for obvious bugs and anomalies, monitoring the node evaluation statistics as well. This helped to detect problems associated with the lack of monotony in the evaluator, for instance. Since it is quite difficult for a human to keep track of depth with rather simple (but appreciated) graphical representation provided, it was expected that algorithms beat the tester, and don't miss obvious threats. The testing also demonstrated that the evaluator performed very well, making even the 2-ply minimax program defend effectively against human threatening sequence plans that were 6-8 ply long. When the alpha-beta algorithm was implemented, it was tested for correctness by setting it to search 2 ply, and running it and minimax simultaneously, giving them the same input, and comparing their decisions, to make sure they match.

The tournament was run by playing each program against each other one, in both move orders; it was driven by a simple shell script (tourney.sh). All results and statistics for each program where output to separate log files, they are enclosed in the appendix. The statistic module calculated, for each game, the arithmetic mean of the move length, the number of moves, and the effective branching factor. These move averages combined in weighted arithmetic averages to calculate average move length for algorithm; and an unweighted geometric average of effective branching factors was used to calculate the average EBF¹. The second metric was made unweighted for two reasons: longer games tended to have lower EBFs (as there were fewer entries towards the end), so weighting would bias the calculation towards lower EBFs; and due to the practical difficulty of calculating geometric means of many large numbers. Further, when calculating average EBF for the game, moves with single option (i.e. BF=1) were excluded as atypical, to avoid skewing the results down too much.

The human testing ensured that the implementations have some level of playing strength; yet there is a potential pitfall with the tournament results, in that the (deterministic) algorithms always play out the same games, so it's hard to judge their true strengths; yet since the relative strength of programs remained fairly consistent throughout the bug fixing, it's likely at least somewhat relevant. Further, the stability of effective branching factors against various other programs suggest that the measurement of them are meaningful.

3 Results.

3.1 Tournament Results

The following is the tournament table:

¹Calculation of one of the entries for minimax's results by the statistics package overflowed, so an approximate value was used instead for one of the data points.

Algorithm\vs.	Spot	Minimax	$\alpha - \beta$	$\alpha - \beta$ with ordering	Points
Spot	x	-/-	-/-	-/-	0
Minimax	+/+	x	+/-	-/0	7
$\alpha - \beta$	+/+	+/-	x	+/-	8
$\alpha - \beta$ with ordering	+/+	0/+	+/-	x	9

2 points were awarded for a win (+), 1 for a tie(0) , 0 for a loss(-). Each cell includes 2 games, corresponding to two possible turn orders. The first corresponds to the algorithm/program on the left going first.

3.2 Summary of Statistical Data

Algorithm	Search depth (ply)	Ave. Move Time (secs)	Ave. Effective Branching Factor
Minimax	2	0.165	47.05
$\alpha - \beta$	3	1.61	20.38
$\alpha - \beta$ with ordering	4+	2.93	8.47

4 Analysis

Probably the most visible (and the most obvious) result is that even in a simple game, but one with a very high branching factor, the exponential explosion of number of moves limits the search depth to very small values. There is probably no way around using relatively simple game search algorithms (although more sophisticated ones than these, such as NegaScout, etc.) in more complex games like chess, which have enormously sophisticated structure, but it seems that Qubic could probably benefit from more specialized algorithms that take advantage of the game's properties to reduce the search tree. Victor Allis's application of DB-Search to efficiently analyze sequences of forced moves, used by him to solve the game, appears to be a confirmation, although it's not quite clear as to how the algorithm could be applied in a tournament-style event, with limited time, except maybe as a tactical module for endgame play.

The difference between strength of play seems somewhat minor; but the only way to really test it would be to have a tournament against many different programs with different algorithms/evaluators, something that is not possible at this time. A big reason, it seems, that the evaluator is giving a great jobs pointing out strategic positional advantages, and thus longer lookahead only helps somewhat.

Of course, the effect of $\alpha - \beta$ on reducing the branching factor is also very impressive. Using the rather simple ordering heuristic of 1-lookahead application of the evaluator, it is able to get close to the square root of the original branching factor; doubling the depth of the search. That, however, is not always enough; as the tie between ordered $\alpha - \beta$ and minimax illustrates — when the ordered $\alpha - \beta$ was tested with a modification permitting it to vary the depth of the search depending on current EBF; and when it went up to 6-ply in some of the mid-game moves, $\alpha - \beta$ was able to achieve a victory. (The code for varying search depth is not included in the solution due to the difficulty of getting good time bounds on behavior). It's likely that quiescence search would have helped, but the real difficulty here is that long-term strategic planning is necessary for a good endgame, but difficult to do without some sort of a playbook; the evaluation function helps find out positional advantages, but that's probably not enough.

The speed of the evaluator is a big factor, too — the most I could produce was 12,000 static evaluations per second, which was not enough to get to even 3-ply with minimax. Using a lower-level language than LISP could be very helpful, particularly since the evaluator algorithm is very bit-op oriented. Increasing the speed by a factor of 10, which seems realistic when writing closer to the hardware, could probably give at least additional 1 ply of lookahead. And since this evaluator is pretty close to what one can have minimally, yet very effective in terms of detecting even positional advantages, it's questionable that a difference one would have any significant effect on the game-playing performance.

Additional algorithmic optimizations are possible, too - transposition tables with 4-ply can probably reduce the branching factor further, and of course so can the usage of more sophisticated algorithms.

A Statistical Information Logs

A.1 Minimax

A.1.1 Vs Spot

```
1. Run time:0.3 Nodes:4033 Effective Branching factor:63.505905
3. Run time:0.29 Nodes:3783 Effective Branching factor:61.506096
5. Run time:0.26 Nodes:3541 Effective Branching factor:59.5063
7. Run time:0.25 Nodes:3307 Effective Branching factor:57.506523
9. Run time:0.23 Nodes:3081 Effective Branching factor:55.506756
11. Run time:0.22 Nodes:2863 Effective Branching factor:53.507008
13. Run time:0.19 Nodes:2653 Effective Branching factor:51.507282
15. Run time:0.18 Nodes:2451 Effective Branching factor:49.507576
Game: Average Move Time:0.24000001 (Weight:8, Weighted:1.9200001) Average EBF:56.32013
```

A.1.2 Vs $\alpha - \beta$

```
1. Run time:0.3 Nodes:4033 Effective Branching factor:63.505905
3. Run time:0.28 Nodes:3783 Effective Branching factor:61.506096
5. Run time:0.27 Nodes:3541 Effective Branching factor:59.5063
7. Run time:0.24 Nodes:3307 Effective Branching factor:57.506523
9. Run time:0.23 Nodes:3081 Effective Branching factor:55.506756
11. Run time:0.21 Nodes:2863 Effective Branching factor:53.507008
13. Run time:0.2 Nodes:2653 Effective Branching factor:51.507282
15. Run time:0.18 Nodes:2451 Effective Branching factor:49.507576
17. Run time:0.18 Nodes:2257 Effective Branching factor:47.507893
Game: Average Move Time:0.23222224 (Weight:9, Weighted:2.0900002) Average EBF:55.26533
```

A.1.3 Vs $\alpha - \beta$ ordered

```
1. Run time:0.3 Nodes:4033 Effective Branching factor:63.505905
3. Run time:0.29 Nodes:3783 Effective Branching factor:61.506096
5. Run time:0.26 Nodes:3541 Effective Branching factor:59.5063
7. Run time:0.25 Nodes:3307 Effective Branching factor:57.506523
9. Run time:0.24 Nodes:3081 Effective Branching factor:55.506756
11. Run time:0.22 Nodes:2863 Effective Branching factor:53.507008
13. Run time:0.2 Nodes:2653 Effective Branching factor:51.507282
15. Run time:0.18 Nodes:2451 Effective Branching factor:49.507576
17. Run time:0.17 Nodes:2257 Effective Branching factor:47.507893
19. Run time:0.16 Nodes:2071 Effective Branching factor:45.50824
21. Run time:0.14 Nodes:1893 Effective Branching factor:43.50862
23. Run time:0.12 Nodes:1723 Effective Branching factor:41.509037
25. Run time:0.11 Nodes:1561 Effective Branching factor:39.50949
27. Run time:0.11 Nodes:1407 Effective Branching factor:37.51
29. Run time:0.09 Nodes:1261 Effective Branching factor:35.510563
```

Game: Average Move Time:0.18933333 (Weight:15, Weighted:2.84) Average EBF:48.73855

A.1.4 At $\alpha - \beta$

2. Run time:0.29 Nodes:3907 Effective Branching factor:62.506
 4. Run time:0.27 Nodes:3661 Effective Branching factor:60.5062
 6. Run time:0.26 Nodes:3423 Effective Branching factor:58.50641
 8. Run time:0.24 Nodes:3193 Effective Branching factor:56.506638
 10. Run time:0.22 Nodes:2971 Effective Branching factor:54.50688
 Game: Average Move Time:0.25599998 (Weight:5, Weighted:1.28) Average EBF:58.437977

A.1.5 At $\alpha - \beta$

2. Run time:0.29 Nodes:3907 Effective Branching factor:62.506
 4. Run time:0.27 Nodes:3661 Effective Branching factor:60.5062
 6. Run time:0.26 Nodes:3423 Effective Branching factor:58.50641
 8. Run time:0.24 Nodes:3193 Effective Branching factor:56.506638
 10. Run time:0.23 Nodes:2971 Effective Branching factor:54.50688
 12. Run time:0.2 Nodes:2757 Effective Branching factor:52.50714
 14. Run time:0.19 Nodes:2551 Effective Branching factor:50.507423
 16. Run time:0.17 Nodes:2353 Effective Branching factor:48.507732
 18. Run time:0.16 Nodes:2163 Effective Branching factor:46.508064
 20. Run time:0.15 Nodes:1981 Effective Branching factor:44.508427
 22. Run time:0.13 Nodes:1807 Effective Branching factor:42.508823
 24. Run time:0.13 Nodes:1641 Effective Branching factor:40.50926
 26. Run time:0.11 Nodes:1483 Effective Branching factor:38.50974
 28. Run time:0.1 Nodes:1333 Effective Branching factor:36.510273
 30. Game: Average Move Time:0.18785714 (Weight:14, Weighted:2.6299999) Average EBF:48.839928

A.1.6 At $\alpha - \beta$ ordered

2. Run time:0.29 Nodes:3907 Effective Branching factor:62.506
 4. Run time:0.27 Nodes:3661 Effective Branching factor:60.5062
 6. Run time:0.25 Nodes:3423 Effective Branching factor:58.50641
 8. Run time:0.24 Nodes:3193 Effective Branching factor:56.506638
 10. Run time:0.22 Nodes:2971 Effective Branching factor:54.50688
 12. Run time:0.2 Nodes:2757 Effective Branching factor:52.50714
 14. Run time:0.19 Nodes:2551 Effective Branching factor:50.507423
 16. Run time:0.17 Nodes:2353 Effective Branching factor:48.507732
 18. Run time:0.16 Nodes:2163 Effective Branching factor:46.508064
 20. Run time:0.15 Nodes:1981 Effective Branching factor:44.508427
 22. Run time:0.14 Nodes:1807 Effective Branching factor:42.508823
 24. Run time:0.13 Nodes:1641 Effective Branching factor:40.50926
 26. Run time:0.11 Nodes:1483 Effective Branching factor:38.50974
 28. Run time:0.1 Nodes:1333 Effective Branching factor:36.510273
 30. Run time:0.09 Nodes:1191 Effective Branching factor:34.510868
 32. Run time:0.08 Nodes:1057 Effective Branching factor:32.511536
 34. Run time:0.07 Nodes:931 Effective Branching factor:30.512293
 36. Run time:0.06 Nodes:813 Effective Branching factor:28.513155

38. Run time:0.05 Nodes:703 Effective Branching factor:26.514147
 40. Run time:0.04 Nodes:601 Effective Branching factor:24.5153
 42. Run time:0.04 Nodes:507 Effective Branching factor:22.51666
 44. Run time:0.03 Nodes:421 Effective Branching factor:20.518284
 46. Run time:0.03 Nodes:343 Effective Branching factor:18.52026
 48. Run time:0.02 Nodes:273 Effective Branching factor:16.52271
 50. Run time:0.02 Nodes:211 Effective Branching factor:14.525839
 52. Run time:0.01 Nodes:157 Effective Branching factor:12.529964
 54. Run time:0.01 Nodes:111 Effective Branching factor:10.535654
 56. Run time:0.0 Nodes:73 Effective Branching factor:8.5440035
 58. Run time:0.01 Nodes:43 Effective Branching factor:6.5574384
 60. Run time:0.0 Nodes:21 Effective Branching factor:4.582576
 62. Run time:0.0 Nodes:7 Effective Branching factor:2.6457512
 64. Run time:0.0 Nodes:1 Effective Branching factor:1.0
 Game: Average Move Time:0.09937499 (Weight:32, Weighted:3.1799996) Average EBF: ∞

A.2 $\alpha - \beta$

A.2.1 Vs Spot

1. Run time:0.62 Nodes:4320 Effective Branching factor:16.286507
 3. Run time:2.17 Nodes:14802 Effective Branching factor:24.553127
 5. Run time:2.19 Nodes:15131 Effective Branching factor:24.733707
 7. Run time:1.49 Nodes:10326 Effective Branching factor:21.775963
 9. Run time:2.59 Nodes:17708 Effective Branching factor:26.064926
 11. Run time:3.5 Nodes:23918 Effective Branching factor:28.812103
 13. Run time:4.78 Nodes:32677 Effective Branching factor:31.97035
 15. Run time:4.97 Nodes:33744 Effective Branching factor:32.314606
 17. Run time:4.97 Nodes:33533 Effective Branching factor:32.24711
 19. Run time:0.31 Nodes:2079 Effective Branching factor:12.762962
 Game: Average Move Time:2.7589998 (Weight:10, Weighted:27.589998) Average EBF:24.195919

A.2.2 Vs Minimax

1. Run time:0.63 Nodes:4320 Effective Branching factor:16.286507
 3. Run time:2.71 Nodes:18685 Effective Branching factor:26.53573
 5. Run time:3.52 Nodes:24208 Effective Branching factor:28.928082
 7. Run time:1.76 Nodes:12178 Effective Branching factor:23.00693
 9. Run time:0.56 Nodes:3762 Effective Branching factor:15.552717
 11. Run time:3.55 Nodes:24422 Effective Branching factor:29.013073
 13. Run time:5.28 Nodes:36181 Effective Branching factor:33.074516
 15. Run time:5.09 Nodes:34987 Effective Branching factor:32.70661
 17. Run time:1.58 Nodes:10728 Effective Branching factor:22.054958
 19. Run time:3.07 Nodes:21058 Effective Branching factor:27.614618
 21. Run time:5.21 Nodes:35675 Effective Branching factor:32.91961
 23. Run time:2.81 Nodes:19234 Effective Branching factor:26.793116
 25. Run time:1.94 Nodes:13288 Effective Branching factor:23.685717
 27. Run time:0.97 Nodes:6724 Effective Branching factor:18.87452
 29. Run time:0.8 Nodes:5486 Effective Branching factor:17.636751

Game: Average Move Time:2.632 (Weight:15, Weighted:39.48) Average EBF:24.263384

A.2.3 Vs $\alpha - \beta$ ordered

1. Run time:0.63 Nodes:4320 Effective Branching factor:16.286507
 3. Run time:1.74 Nodes:11937 Effective Branching factor:22.854149
 5. Run time:3.76 Nodes:25897 Effective Branching factor:29.585789
 7. Run time:3.5 Nodes:24209 Effective Branching factor:28.92848
 9. Run time:2.03 Nodes:14103 Effective Branching factor:24.160383
 11. Run time:2.41 Nodes:17571 Effective Branching factor:25.997534
 13. Run time:1.14 Nodes:7721 Effective Branching factor:19.764744
 15. Run time:0.85 Nodes:5851 Effective Branching factor:18.019526
 17. Run time:0.71 Nodes:4995 Effective Branching factor:17.094057
 19. Run time:1.94 Nodes:13552 Effective Branching factor:23.841549
 21. Run time:1.02 Nodes:7303 Effective Branching factor:19.401432
 23. Run time:1.08 Nodes:7436 Effective Branching factor:19.518501
 25. Run time:0.05 Nodes:316 Effective Branching factor:6.8112845
 Game: Average Move Time:1.6046153 (Weight:13, Weighted:20.859999) Average EBF:19.862923

A.2.4 At Spot

1. Run time:1.02 Nodes:7198 Effective Branching factor:19.307999
 3. Run time:2.22 Nodes:15217 Effective Branching factor:24.780478
 5. Run time:1.62 Nodes:11412 Effective Branching factor:22.514065
 7. Run time:1.48 Nodes:10167 Effective Branching factor:21.663616
 9. Run time:1.46 Nodes:10035 Effective Branching factor:21.569452
 11. Run time:1.69 Nodes:11438 Effective Branching factor:22.53115
 13. Run time:0.02 Nodes:146 Effective Branching factor:5.2656374
 Game: Average Move Time:1.3585714 (Weight:7, Weighted:9.51) Average EBF:17.936153

A.2.5 At minimax

2. Run time:1.13 Nodes:7847 Effective Branching factor:19.87168
 4. Run time:1.36 Nodes:9354 Effective Branching factor:21.07006
 6. Run time:1.18 Nodes:8256 Effective Branching factor:20.211098
 8. Run time:1.69 Nodes:11815 Effective Branching factor:22.776024
 10. Run time:1.81 Nodes:12646 Effective Branching factor:23.29795
 12. Run time:0.87 Nodes:6054 Effective Branching factor:18.225557
 14. Run time:1.08 Nodes:7882 Effective Branching factor:19.901178
 16. Run time:0.1 Nodes:676 Effective Branching factor:8.776383
 Game: Average Move Time:1.1525 (Weight:8, Weighted:9.22) Average EBF:18.594955

A.2.6 At $\alpha - \beta$ ordered

2. Run time:1.12 Nodes:7847 Effective Branching factor:19.87168
 4. Run time:1.36 Nodes:9354 Effective Branching factor:21.07006
 6. Run time:1.02 Nodes:7035 Effective Branching factor:19.16114
 8. Run time:1.68 Nodes:11815 Effective Branching factor:22.776024
 10. Run time:1.35 Nodes:9490 Effective Branching factor:21.171684
 12. Run time:1.93 Nodes:13251 Effective Branching factor:23.663713

14. Run time:1.12 Nodes:7674 Effective Branching factor:19.724558
 16. Run time:1.83 Nodes:12877 Effective Branching factor:23.438953
 18. Run time:0.12 Nodes:780 Effective Branching factor:9.205164
 Game: Average Move Time:1.2811111 (Weight:9, Weighted:11.53) Average EBF:19.401981

A.3 $\alpha - \beta$ ordered

A.3.1 Vs Spot

1. Run time:4.23 Nodes:23069 Effective Branching factor:12.324156
 3. Run time:6.58 Nodes:38907 Effective Branching factor:14.044521
 5. Run time:4.13 Nodes:20975 Effective Branching factor:12.03443
 7. Run time:2.05 Nodes:10493 Effective Branching factor:10.121035
 9. Run time:0.0 Nodes:1 Effective Branching factor:1.0
 Game: Average Move Time:3.398 (Weight:5, Weighted:16.99) Average EBF:12.049767

A.3.2 Vs Minimax

1. Run time:4.24 Nodes:23069 Effective Branching factor:12.324156
 3. Run time:3.76 Nodes:21504 Effective Branching factor:12.1096
 5. Run time:3.73 Nodes:19981 Effective Branching factor:11.889246
 7. Run time:4.19 Nodes:20130 Effective Branching factor:11.911349
 9. Run time:3.62 Nodes:21263 Effective Branching factor:12.075528
 11. Run time:3.23 Nodes:16881 Effective Branching factor:11.398548
 13. Run time:2.89 Nodes:15588 Effective Branching factor:11.173716
 15. Run time:0.87 Nodes:3489 Effective Branching factor:7.685555
 17. Run time:1.94 Nodes:9756 Effective Branching factor:9.938434
 19. Run time:1.86 Nodes:9391 Effective Branching factor:9.844144
 21. Run time:1.18 Nodes:5114 Effective Branching factor:8.4564905
 23. Run time:0.36 Nodes:689 Effective Branching factor:5.123359
 25. Run time:6.37 Nodes:10503 Effective Branching factor:6.371808
 27. Run time:6.29 Nodes:15285 Effective Branching factor:6.8683605
 29. Run time:1.34 Nodes:6694 Effective Branching factor:5.8228707
 31. Run time:1.02 Nodes:4878 Effective Branching factor:5.465732
 33. Run time:0.68 Nodes:3175 Effective Branching factor:5.0158987
 35. Run time:1.18 Nodes:6110 Effective Branching factor:5.7175274
 37. Run time:0.71 Nodes:3407 Effective Branching factor:5.0871487
 39. Run time:3.98 Nodes:18697 Effective Branching factor:7.150796
 41. Run time:1.61 Nodes:3903 Effective Branching factor:5.2273273
 43. Run time:1.5 Nodes:4348 Effective Branching factor:5.3414345
 45. Run time:1.03 Nodes:3190 Effective Branching factor:5.020629
 47. Run time:0.21 Nodes:887 Effective Branching factor:3.886733
 49. Run time:1.12 Nodes:5492 Effective Branching factor:4.2003784
 51. Run time:0.81 Nodes:4262 Effective Branching factor:4.026573
 53. Run time:0.55 Nodes:2715 Effective Branching factor:3.7350376
 55. Run time:0.58 Nodes:2229 Effective Branching factor:3.0081635
 57. Run time:0.31 Nodes:1657 Effective Branching factor:2.525895
 59. Run time:0.04 Nodes:207 Effective Branching factor:2.4321764
 61. Run time:0.01 Nodes:31 Effective Branching factor:2.359611

63. Run time:0.0 Nodes:5 Effective Branching factor:2.236068
 Game: Average Move Time:1.9128126 (Weight:32, Weighted:61.210003) Average EBF:5.9194465

A.3.3 Vs $\alpha - \beta$

1. Run time:4.25 Nodes:23069 Effective Branching factor:12.324156
 3. Run time:3.12 Nodes:17280 Effective Branching factor:11.465314
 5. Run time:3.75 Nodes:18318 Effective Branching factor:11.633744
 7. Run time:1.57 Nodes:7231 Effective Branching factor:9.2214575
 9. Run time:2.39 Nodes:11465 Effective Branching factor:10.3476925
 11. Run time:1.38 Nodes:6310 Effective Branching factor:8.91266
 13. Run time:1.52 Nodes:6196 Effective Branching factor:8.872129
 15. Run time:1.17 Nodes:5151 Effective Branching factor:8.4717455
 17. Run time:0.99 Nodes:4283 Effective Branching factor:8.089786
 19. Run time:0.0 Nodes:1 Effective Branching factor:1.0
 Game: Average Move Time:2.014 (Weight:10, Weighted:20.14) Average EBF:9.821

A.3.4 At Spot

2. Run time:5.27 Nodes:29968 Effective Branching factor:13.157229
 4. Run time:5.09 Nodes:30568 Effective Branching factor:13.222597
 6. Run time:2.46 Nodes:11498 Effective Branching factor:10.35513
 8. Run time:2.93 Nodes:14805 Effective Branching factor:11.030675
 10. Run time:2.56 Nodes:12456 Effective Branching factor:10.564396
 12. Run time:2.46 Nodes:12163 Effective Branching factor:10.501714
 14. Run time:2.68 Nodes:12498 Effective Branching factor:10.57329
 16. Run time:0.01 Nodes:1 Effective Branching factor:1.0
 Game: Average Move Time:2.9325001 (Weight:8, Weighted:23.460001) Average EBF:11.285159

A.3.5 At Minimax

2. Run time:3.49 Nodes:17432 Effective Branching factor:11.490444
 4. Run time:3.29 Nodes:15994 Effective Branching factor:11.245772
 6. Run time:2.37 Nodes:12539 Effective Branching factor:10.58195
 8. Run time:3.19 Nodes:16928 Effective Branching factor:11.406474
 10. Run time:0.96 Nodes:3547 Effective Branching factor:7.7172985
 12. Run time:2.2 Nodes:11969 Effective Branching factor:10.459585
 14. Run time:0.84 Nodes:3126 Effective Branching factor:7.477342
 16. Run time:0.75 Nodes:2767 Effective Branching factor:7.2527432
 18. Run time:13.6 Nodes:30765 Effective Branching factor:7.8997145
 20. Run time:1.93 Nodes:7199 Effective Branching factor:5.90819
 22. Run time:1.21 Nodes:4995 Effective Branching factor:5.4917035
 24. Run time:1.09 Nodes:3719 Effective Branching factor:5.1770844
 26. Run time:0.58 Nodes:1790 Effective Branching factor:4.4727087
 28. Run time:5.96 Nodes:17069 Effective Branching factor:5.074205
 30. Run time:0.01 Nodes:1 Effective Branching factor:1.0
 Game: Average Move Time:2.7646666 (Weight:15, Weighted:41.469997) Average EBF:7.5771337

A.3.6 At $\alpha - \beta$

2. Run time:3.5 Nodes:17432 Effective Branching factor:11.490444
4. Run time:3.29 Nodes:15994 Effective Branching factor:11.245772
6. Run time:2.73 Nodes:13254 Effective Branching factor:10.729679
8. Run time:2.28 Nodes:10669 Effective Branching factor:10.163211
10. Run time:1.39 Nodes:6145 Effective Branching factor:8.853816
12. Run time:1.76 Nodes:7629 Effective Branching factor:9.345809
14. Run time:0.79 Nodes:2796 Effective Branching factor:7.2716722
16. Run time:10.88 Nodes:11098 Effective Branching factor:6.4424186
18. Run time:8.86 Nodes:6871 Effective Branching factor:5.853343
20. Run time:7.43 Nodes:4636 Effective Branching factor:5.4103913
22. Run time:0.93 Nodes:3560 Effective Branching factor:5.1320395
24. Run time:0.26 Nodes:82 Effective Branching factor:2.4141417
Game: Average Move Time:3.675 (Weight:12, Weighted:44.1) Average EBF:7.25975