

CSC282 Fall 2005 Homework #2

1. In certain types of binary trees the base case, instead of being an empty tree, is a tree with a single node of a different type from the rest of the tree. This type of node is called an *external node*. A tree that consists of an external node is called a *leaf*, and it has no subtrees. The other type of node is an *internal node*, and it must have exactly two children. Such binary trees are called *2-trees* – each node has 0 or 2 children.

Show that every 2-tree with n internal nodes has $n + 1$ external nodes.

2. Prove $x \lg(x)$ is convex.

3. Let $T(n)$ be defined for all $n \geq 3$ a power of 3 by the following recurrence:

$$T(n) = 4T(n/3) + 3n + 5$$

$$T(1) = 2.$$

What is the exact solution for T when n is a power of 3?

4. CLRS 4.3-1

5. CLRS 4-4: a, e, i.

6. CLRS 4-6.

7. Let the function M be defined for all powers of 2 and be described by the following recurrence:

$$M(n) = n - 1 + 2M(n/2)$$

$$M(1) = 0.$$

A. What is the asymptotic order of $M(n)$?

B. What is the exact solution for M when n is a power of 2?

8. Suppose W satisfies the following recurrence, where c is a constant:

$$W(n) = cn + W(\lfloor n/2 \rfloor)$$

$$W(1) = 1$$

What is the asymptotic order of $W(n)$?