Department of Computer Science CSC248/448 Fall 2003 Assignment 1 Solutions

Problem 1

(a) We have 2 random variables: a weather variable $W \in \{rain, no - rain\}$ and a friend late variable $FLin\{late, not - late\}$. We have that the probability of rain P(W = rain) = .3, the probability of the friend being late for a movie P(FL = late) = .5 and the probability of rain when the friend is late is P(W = rain|FL = late) = .2. The question is to find the probability that the friend is late when it rains, P(FL = late|W = rain).

By the Bayes rule, $P(FL = late|W = rain) = \frac{P(W = rain|FL = late)P(FL = late)P(FL = late)}{P(W = rain)} = \frac{.2*.5}{.3} = 1/3$

- (b) The random variables are: whether Harry wins a race, $W \in \{win, loose\}$, and whether I drive to the racetrack, $G \in \{drive, walk\}$. We also have that winning the race and the way I get to the racetrack are independent, P(W|G) = P(W). We know that P(W = win) = .3 and P(G = drive) = 0.5. We need to find P(W = win|G = drive) Since W and G are independent, P(W = win|G = drive) = P(W = win) = 0.3.
- (c) We have two random variables, D_1 denoting the ball drawn the first time, and D_2 denoting the ball drawn the second time. Both are over the sample space {blue, cyan}. These variables are independent, because a ball is returned to the urn after the draw and so the second draw has the identical conditions to the first one.
 - (i) $P(D_1 = cyan) = \frac{c}{b+c}$
 - (ii) $P(D_2 = cyan) = \frac{c}{b+c}$ because of the independence assumption
 - (iii) $P(D_1 = cyan, D_2 = cyan) = P(D_1 = cyan) * P(D_2 = cyan) = \frac{c^2}{(b+c)^2}$ because of the independence assumptions
 - (iv) $P(D_1 = cyan | D_2 = cyan) = P(D_1 = cyan) = \frac{c}{b+c}$ because D_1 and D_2 are independent

Problem 2

The variables and the sample space are the same as in problem 1c, but now there is no longer the independence assumption, because the number of balls changes after the first draw.

- (i) $P(D_1 = cyan) = \frac{c}{b+c}$
- (ii) $P(D_2 = cyan) = P(D_2 = cyan, D_1 = cyan) + P(D_2 = cyan, D_1 = blue)$ = $P(D_2 = cyan|D_1 = cyan)P(D_1 = cyan) + P(D_2 = cyan|D_1 = blue)P(D_1 = blue)$ = $\frac{c+d}{b+c+d}\frac{c}{b+c} + \frac{c}{b+c+d}\frac{b}{b+c} = \frac{c}{b+c}$

(iii)
$$P(D_1 = cyan, D_2 = cyan) = P(D_1 = cyan) * P(D_2 = cyan|D_1 = cyan) = \frac{c}{(b+c)} \frac{c+d}{b+c+d}$$

(iv)
$$P(D_1 = cyan | D_2 = cyan) = \frac{P(D_2 = cyan | D_1 = cyan)P(D_1 = cyan)}{P(D_2 = cyan)} = \frac{\frac{c+d}{b+c+d} \frac{c}{b+c}}{\frac{c}{b+c}} = \frac{c+d}{b+c+d}$$
 by the Bayes rule

Problem 3

Something is a probability distribution if it satisfies 3 axioms

- (a) $0 \le P(x) \le 1$ for all x
- **(b)** $\sum_{alli} P(x_i) = 1$
- (c) $P(x \in \{x_1, ..., x_n\}) = \sum_{i=1}^n P(x = x_i)$

We need to prove all these axioms for our function $P(x) = l_1P_1(x) + l_2P_2(x)$ where P_1 and P_2 are probability distributions and thus satisfy the axioms above, and $l_1 + l_2 = 1$ and $l_1 \ge 0$, $l_2 \ge 0$. We have that $P(x) = l_1P_1(x) + l_2P_2(x) \ge 0$ because $l_1, l_2, P_1(x), P_2(x) \ge 0$. Then $P(x) = l_1P_1(x) + l_2P_2(x) \le l_1 * 1 + l_2 * 1 = l_1 + l_2 = 1$ because both P_1 and P_2 satisfy axiom (a).

To prove axiom (b), we have that $\sum_{alli} P(x_i) = \sum_{alli} (l_1 P_1(x_i) + l_2 P_2(x_i)) = \sum_{alli} (l_1 P_1(x_i)) + \sum_{alli} (l_2 P_2(x_i)) = l_1 \sum_{alli} P_1(x_i) + l_2 \sum_{alli} P_2(x_i) = l_1 * 1 + l_2 * 1 = 1$ because both P_1 and P_2 satisfy axiom (b) and $l_1 + l_2 = 1$.

To prove axiom (c), we have that $P(x \in \{x_1, ..., x_n\}) = l_1 P_1(x \in \{x_1, ..., x_n\}) + l_2 P_2(x \in \{x_1, ..., x_n\}) = l_1 \sum_{i=1}^n P_1(x = x_i) + l_2 \sum_{i=1}^n P_2(x = x_i) \sum_{i=1}^n (l_1 P_1(x = x_i) + l_2 P_2(x = x_i)) = \sum_{i=1}^n P(x = x_i)$ because both P_1 and P_2 satisfy axiom (c).