CSC 444 Final Exam

There are 6 questions. Solutions are due by electronic turn-in no later than 11:59pm on Monday, December 20. Upload one or more pdf files with your solutions. The file name should contain your own first and last name, e.g. joe-smith-1.pdf, joe-smith-2.pdf. Do not turn in odt, doc, or any other format files. Your answers may be typed or neatly hand-printed and scanned. If the page is hand-written, it is acceptable to have a few items crossed out, but if the page was extensively revised, make a clean copy before scanning it. You may upload your solutions multiple times. You may refer to your notes and textbooks and other general sources of information. However, you should not discuss the problems or search on the web for solutions to the particular problems. (You may search for general information about a topic, for example, modal logic, resolution, etc.)

1. The assertions:
   - Anyone who loves all people is enlightened.
   - The Buddha loves all living things.
   - People are living things.
   can be translated into FOL as follows:
   \[
   \forall x (\forall y (\text{person}(y) \supset \text{loves}(x, y)) \supset \text{enlightened}(x))
   \forall x (\text{living}(x) \supset \text{loves}(\text{Buddha}, x))
   \forall x (\text{person}(x) \supset \text{living}(x))
   \]
   Create a resolution refutation proof that the Buddha is enlightened:
   \[\text{enlightened(\text{Buddha})}\]
   Be careful to handle conversion to CNF, skolemization, and renaming of variables during resolution properly. Show all details. Make sure it is clear which pairs of literals are being resolved upon. In particular, be very careful in translating the first FOL statement into CNF: this is the kind of nested quantification that many people got wrong on the midterm exam.

2. Consider the following Markov Logic theory:
   - Hard formula: \(\forall x (\neg\text{penguin}(x) \lor \text{bird}(x))\)
   - Soft formula, weight 10: \(\forall x (\neg\text{bird}(x) \lor \text{fly}(x))\)
   - Soft formula, weight 20: \(\forall x (\neg\text{penguin}(x) \lor \neg\text{fly}(x))\)
   - Hard formula: \(\text{penguin(\text{Tweety})}\)
   Calculate \(P(\text{fly(\text{Tweety})})\)
   Show all steps of your calculation. Begin by listing all of the models of the hard formulas, and computing the sum of the weighted ground clauses true in each model.

Next, suppose the following clauses are added to the theory:
   - Soft formula, weight 15: \(\forall x, y (\neg\text{friend}(x, y) \lor \text{fly}(x) \lor \neg\text{fly}(y))\)
   - Hard formula: \(\text{penguin(\text{Fred})}\)
   - Hard formula: \(\text{friend(\text{Tweety, Fred})}\)
Hard formula: $\text{friend}(Fred, Tweety)$
Note that the soft formula above is equivalent to
$\forall x, y (\text{friend}(x, y) \lor \neg \text{fly}(x)) \supset \neg \text{fly}(y)$
Calculate $P(\text{fly}(Tweety))$ and $P(\text{fly}(Tweety) \land \text{fly}(Fred))$. Show all steps.

3. Consider the follow propositional theory:
$\neg \text{healthy} \supset (\text{bacterialInfection} \lor \text{viralInfection})$
$\text{takingAntibiotics} \supset \neg \text{BacterialInfection}$
$\text{fever} \supset \neg \text{healthy}$
Compute the Horn LUB and all of the Horn GLBs. Show all of your work. Simplify the bounds as much as possible.

Next, use the bounds to try to determine the answers to the following two queries.
For each, show how using the bounds would return Yes, No, or Unknown.
$\text{bacterialInfection} \lor \text{takingAntibiotics}$
$(\text{takingAntibiotics} \land \text{fever}) \supset \text{viralInfection}$

4. Write a Kripke structure with 2 agents, John and Alice, 3 states, $w_0$, $w_1$, and $w_2$, and the single proposition $S$, which means “it is snowing”, such that the following statements are true in $w_0$:
- John believes it is snowing.
- Alice believes it is snowing.
- John believes that Alice believes it is not snowing.
- John believes that Alice believes that John believes it is snowing.
Your structure should respect the property of belief that if an agent believes something, then he believes that he believes that thing. For example, since John believes it is snowing, then John believes that John believes it is snowing.

5. Consider the following diagnosis problem. The background theory is:
$(\text{turnKey} \land \neg \text{Ab(battery)} \land \neg \text{Ab(gas)}) \supset \text{engineStarts}$
$\text{Ab(gas)} \supset \neg \text{engineRuns}$
$\neg \text{engineRuns} \supset \neg \text{engineStarts}$
$\text{turnKey}$
The observation is $\neg \text{engineStarts}$. The abducibles are the Ab predicates. Compute all of the abductive diagnoses and all of the consistency-based diagnoses. Show all steps of your work.

6. Consider a SAT-modulo theory that contains a proposition $p$ that is supposed to mean “$x = y + 1$” for some integer valued variables $x$ and $y$. Suppose we are doing an eager circuit based encoding, and will represent $x$ and $y$ as 2-bit numbers, using the Boolean propositions $X1, X0, Y1, Y0$, where $X1$ is the high-order bit of $x$, $X0$ is the low-order bit of $x$, etc.

Write a formula in proposition logic that asserts “$p$ if and only if $x = y + 1$”.
Next, convert this formula to CNF.