Question 1. [50 points] Define what it means for $A \leq_m B$ (i.e., give the definition of (recursive) many-one reductions). Note: Give the definition from our course slides (not some obscure, seemingly different definition that happens to capture the same relation).

Answer: We say $A \leq_m B$ iff $(\exists$ recursive function $f)(\forall x)[x \in A \iff f(x) \in B]$. 

Question 2. [50 points] Define what it means for $A \leq_m B$. Let $A = \{x110x \mid x \in HP\}$. Prove that $A$ is undecidable by explicitly giving a recursive many-one reduction from HP to $A$ (which includes of course arguing that the function you give is indeed a recursive many-one reduction from HP to $A$. Recall that HP = $\{x \mid x \in L(M_x)\}$ or, equivalently, $\{x \mid M_x \text{ on input } x \text{ accepts}\}$. (Of course, make sure to give a reduction from HP to $A$ and not the other way around.) Note: $x110x$ denotes $x$ followed by 110 followed by $x$, e.g., if $x$ is 1101 then $x110x$ is 110111011101.

Answer: Our recursive reduction will be the function $f(x) = x110x$. Clearly, this is a recursive function. If $x \in HP$, then by the definition of $A$ it holds that $x110x$ (which is $f(x)$!) is in $A$. If $x \not\in HP$, then by the definition of $A$ it holds that $x110x$ (which is $f(x)$!) is not in $A$. So, regarding our recursive function $f$, it holds that for each $x$, $x \in HP$ iff $f(x) \in A$. Thus we have shown that $HP \leq_m A$. 