Question 1. [100 points] You may do your choice of either (a) or (b). You should NOT do both. Follow exactly the instructions for whichever one of those you do. (If you need room for scrap work—since the instructions are very specific about your answers and what should and should not be in your answer—you may use the back of your paper; we will ignore the back of your paper, and will grade only the front of your paper.)

(a) Give an RE set $C$ and a coRE set $D$ such that $CD$ (i.e., $\{xy \mid x \in C \text{ and } y \in D\}$) equals $\Sigma^*$. For this problem, rather unusually, I am not asking you to include a proof of anything—your answer should just give $C$ and $D$, and that is all your answer should present. (Big hint: For example, the solution that Dan put up on Monday (to the homework problem asking you to show that there exist two nonrecursive sets whose concatenation is recursive) in fact perfectly solves this problem! So if you give his two sets, and get right which is the one to here call $C$ and which is the one to here call $D$, you’ll have given a perfect answer.)

(b) Give a function $\sigma$ such that $\sigma$ is a recursive function that many-one reduces $HP$ to $L$, where always $HP = \{x \mid M_x(x) \text{ accepts}\}$ and $L = \{i \mid L(M_i) \text{ contains at least 123 strings and at most 127 strings}\}$. For this problem, rather unusually, I am not asking you to include a proof of anything—your answer should simply give a function $\sigma$ that has the right properties (but do not give a proof of that). (Hint: This is basically the same as the second-to-last problem on HW Set 2, except the two constants here are different than the ones there.)

ANSWERS:

(a): $C = HP \cup \{\epsilon\}$, $D = \overline{HP} \cup \{\epsilon\}$.

(b): $\sigma(i) = j$, where $j$ is the index of a TM that on input $z$ does the following: If $z$ is not one of the 123 lexicographically smallest strings then reject, otherwise immersively simulate $M_i(i)$. 