Question 1. [100 points] Define the lacing of two languages $A$ and $B$ as $\text{licing}(A, B) = \{xyx \mid x \in A \text{ and } y \in B\}$. Prove that if $A$ and $B$ are r.e. sets, then $\text{licing}(A, B)$ is an r.e. set. (Hint: Be careful not to get tripped up with any issue related to the empty string, or with any issue related to your not coordinating to make sure that in $xyx$ it is the same “$x$” in both places.)

Answer Sketch:

Let $A$, $B$ be two arbitrary RE languages and let $M_A$ and $M_B$ be enumerating TMs that enumerate them. We will construct an enumerating TM $C$ that enumerates $\text{licing}(A, B)$.

Our enumerating TM $C$ will go one ply after another, forever. During the $k$th ply, it will simulate both $M_A$ and $M_B$ for $k$ steps, will keep track of the strings each enumerates in that time, and then for each $a$ that was enumerated by $M_A$ and each $b$ that was enumerated by $M_B$, it will enumerate $aba$.

Our construction ensures that $E(C) = \text{licing}(A, B)$ (since this process enumerates all $xyx$ with $x$ from $A$ and $y$ from $B$, and enumerates nothing else), so $\text{licing}(A, B)$ is always RE when $A$ and $B$ are RE.