1. [40 pts] We have seen in class that if there exists a sparse set that is $\leq^p_m$-hard for co-NP, then $P = NP$. Define what is a sparse set.

2. [30 pts] Theorem 1.2 (Hem-Ogi) states that:

   \[ \text{if there is a tally set } T \subseteq 1^* \text{ that is } \leq^p_m \text{-hard for NP, then } P = NP. \]

   The proof of this theorem assumes that there exists a deterministic polynomial-time function $g$ such that $\forall f \in SAT \text{ if and only if } g(f) \in T$.

   In the proof of this theorem that is in the book (and that we went over in class), there are two different types of “pruning” actions we take at each layer (or level, or depth) in the “tree” of formulas that we are pruning, level-by-level. Describe each of these two types of pruning.

3. [10 pts each] Answer the following True/False questions:

   (a) NP has sparse complete sets only if $P = NP$. _________

   (b) A boolean formula having at least one variable is satisfiable if and only if either it is satisfiable with its first variable set to False or it is satisfiable with its first variable set to True. _________

   (c) NP has sparse complete sets if $P=NP$. _________