Question 1.

a) Let $G$ be arbitrary tournament graph. Invert all the edges of $G$. By Theorem 3.1, in the resulting graph $G'$, there exists a small subset $H$ of players such that all players not in $H$ defeat at least one member of $H$. It is easy to check that this set $H$ satisfies the required conditions in $G$.

b) Consider the following selector $f$ for empty set. Fix any three distinct strings $x, y$ and $z$ and let $f(x, y) = x$, $f(y, z) = y$, $f(x, z) = z$, and for all other inputs $f$ outputs the lexicographically smallest of two arguments. $f(f(x, y), z) = z$, while $f(x, f(y, z)) = x$, so $f$ is not associative.

Question 2.

a) For each natural $n$, consider an advice string having $|\Sigma|^n$ bits such that $i$th bit of advice is set to one iff the $i$th string of length $n$ is a member of SAT.

b) We can use, at each length, a one bit advice string, indicating whether the only potential string of that length $n$, namely $1^n$ is in L or not.

Question 3. Let $L$ be a P-selective NP-complete set, and $f$ a P-selector for it. Let $g$ be a many-one reduction from SAT to $L$. For any boolean formula $F$, consider the self-reducibility tree for $F$. Then from any node $v$ in the tree, let $v_0$ and $v_1$ be the children of $v$. Then if $v$ is in SAT, at least one of $v_0$, $v_1$ are in SAT, so $f(g(v_0), g(v_1))$ is in L and so (recalling that this assumes $v$ is in SAT): if $f(g(v_0), g(v_1)) = g(v_0)$, then $v_0 \in SAT$ and otherwise $v_1 \in SAT$. So if $F$ is in SAT, then repeatedly using $f$, as just described, to select a child will give a satisfying assignment of $F$. So SAT can be decided in polynomial time, and $P=NP$. 