Question 1.
   a) We claim that in each k-tournament, there exists a subset $H$ of players such that
      I) $|H| \leq \lfloor \log(k + 1) \rfloor$, and
      II) Every player not from $H$ loses to at least one player from $H$.
      Prove this not by an explicit repeated reduction of graph size (as in Theorem 3.1), but by assuming that Theorem 3.1 holds and using it and, perhaps, some additional graph manipulations/constructions.
   b) Prove or refute the following claim: all selector functions are associative (in the sense that $f(f(x, y), z) = f(x, f(y, z))$).

Question 2.
   a) Prove that $\text{SAT} \in \mathcal{P}/\text{exp}$ where $\mathcal{P}/\text{exp}$ is the class of exponential functions.
   b) Prove that $\forall L \in \mathcal{TALLY}, L \in \mathcal{P}/\text{const}$, where $\mathcal{P}/\text{const}$ is the class of all $f(n)$, such that $f(n) = O(1)$.

Question 3. Show that if there exists an NP-complete, P-selective language $L$, then $P = NP$. (Hint: Use the self-reducibility of SAT).