In the proof of $P \notin \text{NP}/n$ we constructed a language $L$ which satisfies 3 conditions:

1. $L \subseteq \Sigma^{l_0} \cup \Sigma^{l_1} \cup \Sigma^{l_2} \cup \ldots$. That is, all strings in $L$ have lengths from the set $Q$. (Note that we define $Q = \{l_0, l_1, l_2, \ldots\}$ where $l_0 = 2$, and for each $i \geq 1$, $l_i = 2^{2^{l_{i-1}}}$)

2. For each $x$ and $y$, if $|x| = |y|$ and $x \leq_{lex} y$ and $y \in L$, then $x \in L$.

3. $L \in \text{DTIME}[2^{2^n}]$.

The proof of the theorem is based on two claims:

Claim 3.14 tells us that any set $L$ which satisfies the 3 conditions is semi-feasible.

Claim 3.15 tells us that there exists a set $L$ which satisfies the 3 conditions, but $L$ is not a $\text{NP}/n$ set.

——House is not a home without Lane——

Now please answer two questions about what you’ve learnt in today’s lecture.

1. (60 points, 20 for each) Recall that in the proof of Claim 3.14 we constructed such a selector function $f$, please help finish the construction (there are 3 blanks).

$$
 f(x, y) = \begin{cases} 
 x & \text{if } |y| \notin Q, \\
 y & \text{if } |x| \notin Q \land |y| \in Q, \\
 \min\{x, y\} & \text{if } |x| \in Q \land |y| \in Q \land \\
 & \text{if } |x| \in Q \land |y| \in Q \land |x| \neq |y| \land \min\{x, y\} \in L, \\
 & \text{if } |x| \in Q \land |y| \in Q \land |x| \neq |y| \land \min\{x, y\} \notin L.
\end{cases}
$$
2. (40 points) In the proof of Claim 3.15 ($L \not\in \text{NP}/n$) we used diagonalization. In that proof, at stage $l$, if $l \in Q$ we allowed our simulations to run for at most $2^{2^l}$ steps. Why do you think this constraint is required during the construction of $L^{=l}$? (For example, if we dropped that aspect, would we have severe problems regarding one of the items 1/2/3 from the top of this page, and if so, which one?)