

Homework 2

CSC 280

Due: 1/27/2010

1. Consider the following statement:
Every sufficiently large integer n in the set A can be written as the product of two integers p and q such that p is greater than one and np^k (for every integer $k > 0$) is an integer in A .
 - a) State the statement in first order logic.
 - b) Find the negation of this formula.
 - c) State the negation in natural language form.
 - d) For each of the following cases, specify whether the statement is true or not. Why or why not?
 - d1) $A = \{1, 2, \dots, 10^{1000}\} \cup \{2^n 3^n \mid n \in \mathbb{Z} \ \& \ n > 1000\}$
 - d2) $A = \{1, 2, \dots, 10^{1000}\} \cup \{3^m 5^n \mid n, m \in \mathbb{Z} \ \& \ n, m > 1000\}$
 - d3) $A = \{1, 2, \dots, 10^{1000}\} \cup \{2^n \mid n \in \mathbb{Z} \ \& \ n \text{ is even} \ \& \ n > 1000\}$
 - d4) $A = \{3^n \mid n \in \mathbb{Z} \ \& \ n > 1000\} \cup \{5^n \mid n \in \mathbb{Z} \ \& \ n > 1000\}$
2. Consider the relation ϕ (empty set) over some set A . Determine whether this relation is a) reflexive b) irreflexive c) symmetric d) asymmetric e) anti-symmetric f) transitive g) partial order h) total order. Why or why not?
3. Consider the function $f: A \rightarrow A$ such that $\forall x \in A, f(f(x)) = x$. Prove that f is a bijection.
4. Consider the set $A = \{0, 1, 2, 3, 4\}$. How many distinct
 - a) relations b) reflexive relations c) symmetric relations d) partial functions e) onto functions f) bijections g) unary operators h) binary operatorscan be defined over A ?
5. $G = (V, E)$ is a tournament in which no two vertices have the same out-degree. Prove that $E \cup \{(v, v) \mid v \in V\}$ is a total order over V .