1. Consider the following statement:

Every sufficiently large integer \( n \) in the set \( A \) can be written as the product of two integers \( p \) and \( q \) such that \( p \) is greater than one and \( np^k \) (for every integer \( k > 0 \)) is an integer in \( A \).

a) State the statement in first order logic.

b) Find the negation of this formula.

c) State the negation in natural language form.

d) For each of the following cases, specify whether the statement is true or not. Why or why not?

\[
d1) A = \{1, 2, \ldots 10^{1000}\} \cup \{2^n3^n \mid n \in \mathbb{Z} \& n > 1000\} \\
d2) A = \{1, 2, \ldots 10^{1000}\} \cup \{3^m5^n \mid n, m \in \mathbb{Z} \& n, m > 1000\} \\
d3) A = \{1, 2, \ldots 10^{1000}\} \cup \{2^n \mid n \in \mathbb{Z} \& n \text{ is even} \& n > 1000\} \\
d4) A = \{3^n \mid n \in \mathbb{Z} \& n > 1000\} \cup \{5^n \mid n \in \mathbb{Z} \& n > 1000\}
\]

2. Consider the relation \( \phi \) (empty set) over some set \( A \).

Determine whether this relation is a) reflexive b) irreflexive c) symmetric d) asymmetric e) anti-symmetric f) transitive g) partial order h) total order. Why or why not?

3. Consider the function \( f: A \rightarrow A \) such that \( \forall x \in A, f(f(x)) = x \). Prove that \( f \) is a bijection.

4. Consider the set \( A = \{0, 1, 2, 3, 4\} \). How many distinct

a) relations b) reflexive relations c) symmetric relations d) partial functions e) onto functions f) bijections g) unary operators h) binary operators can be defined over \( A \)?

5. \( G = (V, E) \) is a tournament in which no two vertices have the same out-degree. Prove that \( E \cup \{(v, v) \mid v \in V\} \) is a total order over \( V \).