Exercises: Mathematical Preliminaries
Due 22 September 1998

CSC 172

August 24, 1998

Read the handout on "Mathematical Preliminaries".

1. How many different 5-card draw poker hands may be dealt from a 52 card deck?

2. How many different 5-card draw poker hands may be dealt from a 54 card deck, one with two identical jokers?

3. If you have an unfair coin with probability p of coming up heads, with p not necessarily equal to 1/2, and thus $q = 1 - p$ probability of coming up tails, what is the probability of tossing the coin n times and getting k heads?

4. Explain the fact that $11^4 = 14641$ in terms of Pascal's triangle.

5. What is the relation between $a^{log_c n}$ and $n^{log_c a}$? Prove it.

6. Prove that F_{n+1} and F_n are relatively prime. This is easy once you prove (by induction) that with F_n a Fibonacci number, $F_{n+1}F_{n-1} - F_n^2 = (-1)^n$.

7. Express $\log_{10} x$ in terms of $\log_2 x$ and $\log_2 10$.

8. If n is a 14-digit integer, will the value of n fit in a computer word with a capacity of 47 bits plus sign?

9. A prime number is an integer greater than one which has no exact divisors other than 1 and itself. Using this definition and mathematical induction, prove that every positive integer greater than one may be written as a product of prime numbers. (Hint: don't "induce" by adding 1 to n. Rather start your proof out: "If n is prime it is definitely the product of itself and 1. If it isn't prime...".