

A Less trivial resolution proof

1. No one likes anyone of whom he is afraid
 $\forall x, y. A(x, y) \Rightarrow \neg L(x, y)$ (domain: people)
2. No one likes anyone who does not like him
 $\forall x, y. \neg L(y, x) \Rightarrow \neg L(x, y)$
3. All paranoids are afraid of everyone except their friends
 $H(x, y)$: x has friend y
 $\forall x, y. (P(x) \wedge \neg H(x, y)) \Rightarrow A(x, y)$
4. There is a paranoid, and all of his friends are also paranoid
 $\exists x. P(x) \wedge \forall y. H(x, y) \Rightarrow P(y)$

Conclusion:

There is someone whom all non-paranoids dislike

$$\exists x \forall y. \neg P(y) \Rightarrow \neg L(y, x)$$

Denial

$$\neg \exists x \forall y. \neg P(y) \Rightarrow \neg L(y, x)$$

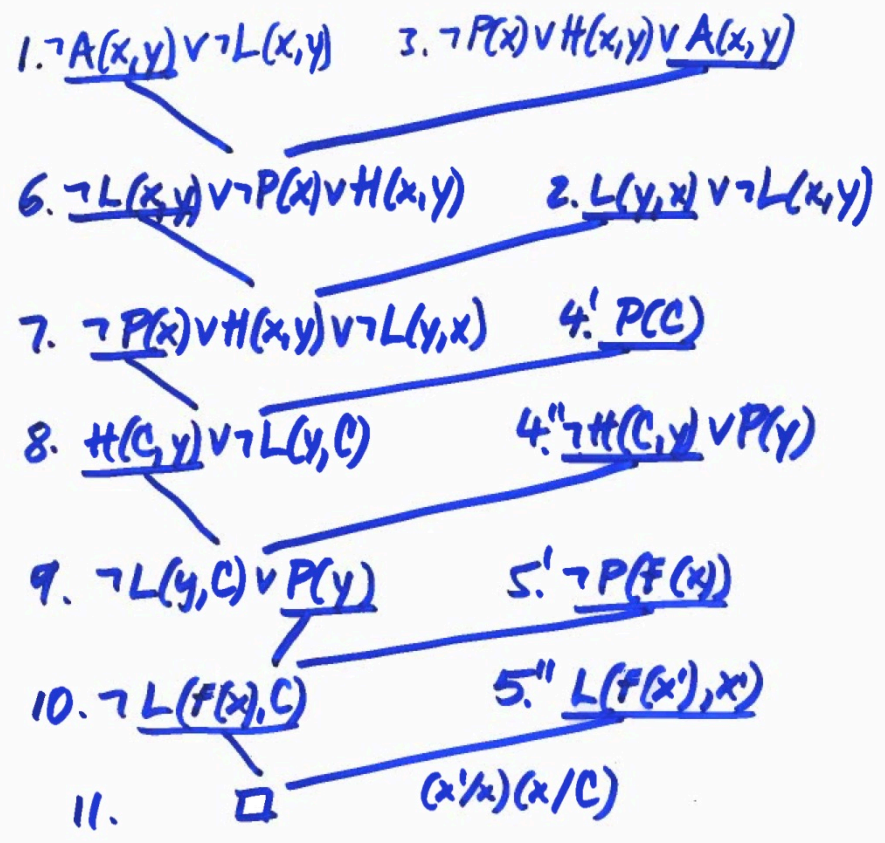
Intuitive argument:

- From 3, all paranoids are afraid of everyone except their friends, so by 1 they don't like anyone except their friends, so by 2, everyone but a paranoid's friends dislike him
- But by 4 there is a paranoid, & so by the previous argument all but his friends dislike him; and since all his friends are paranoids, all non-paranoids dislike him

So, use 3, 1, 2, 4, 5

Clause form

1. $\neg A(x, y) \vee \neg L(x, y)$
2. $L(y, x) \vee \neg L(x, y)$
3. $\neg P(x) \vee H(x, y) \vee A(x, y)$
- 4'. $P(c)$
- 4". $\neg H(c, y) \vee P(y)$
5. $\forall x \exists y. \neg (P(y) \vee \neg L(y, x))$
 \rightsquigarrow 5'. $\neg P(f(x))$
 5". $L(f(x), x)$



Corads: Study the mathematical example (group theory) in the notes.

Finding the m.g.u. of (τ_1, \dots, τ_n) , $(\tau'_1, \dots, \tau'_n)$

1. Standardize apart
2. $mgu := ()$
3. Scan the lists in parallel, left to right
 - a. If end of lists is reached, return mgu (i.e., successful exit, even if $mgu = ()$)
 - b. If current symbols identical, continue at (a) with the next pair (i.e., advance pointers)
 - c. If neither symbol is a variable, return FAIL
 - d. One symbol is a variable, say v , and the other is the beginning of some term τ (a variable, constant, or functional expression).
 If v occurs in τ , return FAIL "occurs-check"
 else (i) push (v/τ) onto mgu
 (ii) make this substitution throughout both lists, from the current pointers forward
 (iii) move the pointers over the (now identical) τ terms

e.g., $(x, F(x)), (y, y)$ can't be unified

Unification example

$$P(s, g(s), t, h(s, t), u, k(s, t, u))$$

$$\neg P(v, w, m(w), x, n(w, x), y)$$

Factoring example

$$P(y, x, A) \vee P(F(y), z, z) \vee P(F(A), B, x)$$

Formal statement of resolution (narrow)

$$\frac{\pi(\sigma_1, \dots, \sigma_k) \vee \alpha_1 \vee \dots \vee \alpha_m, \neg \pi(\tau_1, \dots, \tau_k) \vee \beta_1 \vee \dots \vee \beta_n}{\alpha'_1 \vee \dots \vee \alpha'_m \vee \beta'_1 \vee \dots \vee \beta'_n}$$

where $(\sigma_1, \dots, \sigma_k), (\tau_1, \dots, \tau_k)$ are unifiable with mgu δ ;
 $\alpha'_i = \alpha_i \delta, \beta'_j = \beta_j \delta$, etc.

$\pi(\dots), \neg \pi(\dots)$ need not be initial

Deductive Problem Solving & Q.A.

Cordell Green (IJCAI '69, Mach. Int. '69):

To solve a problem or answer a question,
 1. prove that a solution/answer exists;
 2. extract solution/answer from variable bindings of the proof.

Yes-no questions

Make "parallel" attempts to prove or disprove the proposition in question
 \leftrightarrow yes, no

$\varphi?$ \longrightarrow refute $\neg \varphi \rightarrow$ yes
 \searrow refute $\varphi \rightarrow$ no

Reconsider Snoopy question: Does some x bark?

Wh- Questions ("fill in the blank(s)")

E.g.,

- 1. $\forall x. At(Bob, x) \Rightarrow At(Carol, x)$ Carol is where-
ever Bob is
- 2. $At(Bob, Dance)$ Bob is at the dance

Q: Where is Carol?

Prove: There exists a ^{thing} place where Carol is

$\exists x At(Carol, x)$

Denial: $\neg \exists x At(Carol, x)$

3. i.e., $\forall x. \neg At(Carol, x)$

Refutation & Ans - extraction

- 1. $\neg At(Bob, x) \vee At(Carol, x)$ given
- 2. $At(Bob, Dance)$ given
- 3. $\neg At(Carol, x) \vee \underline{Ans(x)}$ denial
- 4. $At(Carol, Dance)$ $r[1, 2]$
- 5. $\square \vee \underline{Ans(Dance)}$ $r[3, 4]$

Good answers, bad answers

- E.g., 1. $\forall x \exists y. Works-for(x, y)$ Everyone works for someone
- 2. $Works-for(Bob, Carol)$ Bob works for Carol

Q: $\exists x. Works-for(Bob, x)$ Who does Bob work for?

Denial: $\forall x. \neg Works-for(Bob, x)$

First proof

- 1. $Works-for(x, f(x))$ given — employer-of
- 2. $Works-for(Bob, Carol)$ given
- 3. $\neg Works-for(Bob, x) \vee \underline{Ans(x)}$ denial
- 4. $\square \vee \underline{Ans(f(Bob))}$ $r[1, 2]$
Bob works for his employer 😊

Second proof

- 4' $\square \vee \underline{Ans(Carol)}$ $r[2, 2]$
better!

Generally: try to get Skolem-free answers by preferring proof steps that don't substitute Skolem constants/functions into the answer.
But: there may only be Skolem answers — e.g., "Whom does every non-paranoid dislike?"