

# Towards a Consistent Logical Framework for Ontological Analysis

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**Abstract:** In their framework for ontological analysis, Guarino and Welty provide a number of insights that are useful for guiding the design of taxonomic hierarchies. However, the formal statements of these insights as logical schemata are flawed in a number of ways, including inconsistent notation that makes the intended semantics of the logic unclear, false claims of logical consequence, and definitions that provably result in the triviality of some of their property features. This paper makes a negative contribution, by demonstrating these flaws in a rigorous way, but also makes a positive contribution wherever possible, by identifying the underlying intuitions that the faulty definitions were intended to capture, and attempting to formalize those intuitions in a more accurate way.

**Categories & Descriptors:** I.2.4 [Knowledge Representation Formalisms and Methods]: Predicate logic

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## 1. Introduction

In a frequently-cited series of papers [2; 3; 4], Guarino and Welty have developed a system for describing and classifying properties, in support of a principled methodology for building ontologies. They define a number of meta-properties that can be used to classify properties, and they state constraints on which kinds of properties may subsume which other kinds in an ontology. For example, one of the meta-properties is *rigidity*: a property is rigid if every instance of that property has the property necessarily, and a property is anti-rigid if no instance of the property has the property necessarily. The notion of rigidity is brought to bear on the task of building ontologies by the constraint that an anti-rigid property can't subsume a rigid one.

It is clear that many of the ontologies available today were not constructed with precise definitions of taxonomic relations in mind, and the resulting confusion both limits the ontologies' practical usefulness in computational systems, and makes them hard for humans to understand and compare. Guarino and Welty's aim is to improve the

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quality of ontologies by bringing more discipline to the process of constructing them. At first glance, the work may seem to be a step in this direction, because the meta-properties and constraints are presented in the language of mathematical logic, a tool perfectly suited for bringing discipline to the solution of a problem. Unfortunately, a closer look reveals that this tool has not been put to its proper use. The logical formulas and schemata are merely attempts to express certain intuitions symbolically, and have not been subjected to the formal interpretation and validation that logic allows. We will see that, in fact, the mathematical framework is flawed in a number of important ways.

The value of Guarino and Welty’s underlying intuitions as guides for the construction of ontologies is not at issue in this paper. The problem is that, superficial appearances of formality notwithstanding, these intuitions have not yet been fully developed into precise definitions. Given that the very purpose of their work is to bring rigor to a process formerly guided by insufficiently precise intuitions, it is crucial for the success of that work that their own intuitions be developed into a fully precise and correct logical theory.

This paper will point out a number of flaws in Guarino and Welty’s logical system. While each of these flaws will be demonstrated by a rigorous argument, the emphasis will be not on technical details (indeed, all of the proofs are quite simple and straightforward), but rather on the ways in which their logical framework fails to capture the intended concepts and intuitions, and therefore cannot serve the purpose for which it was intended. Where possible, revisions that bring the logical framework in line with intuition are suggested.

## 2. Rigidity

In [3], rigidity and the associated concepts of non-rigidity and anti-rigidity are defined as follows:

- A *rigid property* is a property that is essential to *all* its instances, *i.e.* a property  $\phi$  such that:  $\forall x \phi(x) \rightarrow \Box \phi(x)$  [3, Definition 1].<sup>1</sup>
- A *non-rigid property* is a property that is not essential to *some* of its instances, *i.e.*  $\exists x \phi(x) \wedge \neg \Box \phi(x)$  [3, Definition 2].
- An *anti-rigid property* is a property that is not essential to *all* its instances, *i.e.*  $\forall x \phi(x) \rightarrow \neg \Box \phi(x)$  [3, Definition 3].

Incidentally, note that the schema for anti-rigidity is somewhat misleading: it is logically equivalent to the simpler schema  $\forall x \neg \Box \phi(x)$ . Stating it in the form of a conditional would seem on the surface to imply that the consequent could be false if the antecedent were false, but that is not the case.

These definitions bear comment because they differ from the standard use of the word ‘rigid’ in the modal logic literature. Ordinarily, the word is used to describe a term, and means that the term denotes the same individual in all possible worlds. The

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<sup>1</sup> Rigidity is a second-order property: it is something that is predicated of properties. Following Guarino and Welty, in this paper we will deal with second-order notions in the manner seen above: the first-order part of the definition is stated in first-order logic, and the second-order quantification is expressed in English. For a more rigorous treatment of the meta-properties, a higher-order logic would be useful.

straightforward extension of this concept to predicates would be that a predicate is rigid if it denotes the same set in all possible worlds. This is expressed by the schema

$$\forall x \diamond \phi(x) \rightarrow \Box \phi(x).$$

Guarino and Welty's definition is weaker, *i.e.* more inclusive, than the standard one. This deviation from standard (and more intuitive) terminology may simply be an oversight, since none of Guarino and Welty's results seem to depend on it. The definition of non-rigidity is dependent on that of rigidity; if we replace Guarino and Welty's definition of rigidity with the one given above, then the analogous schema for non-rigidity is  $\exists x \diamond \phi(x) \wedge \neg \Box \phi(x)$ . The schema  $\forall x \neg \Box \phi(x)$  still describes anti-rigidity, regardless of which definition of rigidity is used.

Guarino and Welty state that if  $\phi$  is anti-rigid and  $\psi$  is rigid then  $\phi$  can't subsume  $\psi$  [3, formula 6], where " $\phi$  subsumes  $\psi$ " is defined as  $\forall x \psi(x) \rightarrow \phi(x)$  [3, formula 1]. They claim that this follows directly from the definitions of rigidity and anti-rigidity. This claim is simply false, as is demonstrated by the following model construction.

**THEOREM 1.** *It is possible for an anti-rigid property to subsume a rigid one, given Guarino and Welty's definitions of rigid, anti-rigid, and subsumption.*

**PROOF.** Let  $M$  be a model with two worlds,  $w_1$  and  $w_2$ , and one individual  $i$ . Let the denotation of  $\phi$  be  $\{i\}$  in  $w_1$  and the empty set in  $w_2$ , and let the denotation of  $\psi$  be  $\{i\}$  in both  $w_1$  and  $w_2$ . In this model, designating  $w_1$  as the actual world,

- $\phi$  is anti-rigid: for every individual (namely  $i$ ) of which  $\phi$  is true in  $w_1$ , there is a world (namely  $w_2$ ) in which  $\phi$  is false of that individual;
- $\psi$  is rigid: for every individual (namely  $i$ ) of which  $\psi$  is true in  $w_1$ ,  $\psi$  is true of that individual in all possible worlds;
- $\phi$  subsumes  $\psi$ : for every individual (namely  $i$ ) of which  $\phi$  is true in  $w_1$ ,  $\psi$  is true in  $w_1$ .

□

All three conditions also hold if Guarino and Welty's definition of rigidity is replaced with the standard one, so Theorem 1 has the following corollary:

**THEOREM 2.** *It is possible for an anti-rigid property to subsume a rigid one, given Guarino and Welty's definition of subsumption and anti-rigidity, and defining rigidity to mean denoting the same thing in all possible worlds.*

The proposed constraint is still false even under the constraint that all properties be "discriminating," *i.e.* non-trivial, according to the definition given in [1]:

$$P \text{ is a discriminating property if } \diamond \exists x P(x) \wedge \diamond \exists x \neg P(x).$$

**THEOREM 3.** *It is possible for an anti-rigid, discriminating property to subsume a rigid, discriminating property, given Guarino and Welty's definitions of rigid, anti-rigid, discriminating, and subsumption.*

**PROOF.** Begin with the model constructed in the proof of Theorem 1; add a second individual  $j$  to the domain, but don't add it to the denotation of either predicate. then both  $\phi$  and  $\psi$  are discriminating,  $\phi$  is still anti-rigid,  $\psi$  is still rigid, and  $\phi$  still subsumes  $\psi$ . □

Guarino and Welty apparently have the intuition that an anti-rigid property can't subsume a rigid one. Since this is actually false under the definitions they give for "rigid," "anti-rigid," and "subsumes," it makes sense to ask whether these definitions fail to reflect their intuitions in some way. In fact, there is a plausible explanation: if " $\phi$  subsumes  $\psi$ " is redefined as  $\Box\forall x \psi(x) \rightarrow \phi(x)$ , then it is true that an anti-rigid property can't subsume a rigid one:

**THEOREM 4.** *An anti-rigid property can't subsume a rigid one necessarily, given either definition of rigid.*

The proof of this theorem is trivial.

It seems, therefore, that this stronger definition of subsumption may have been what Guarino and Welty had in mind. It is not an unreasonable definition, but it is different from the one they have stated, in a way that has consequences for the construction of ontologies.

### 3. Necessity, Existence, and Time

As we have seen, the meta-property of rigidity is described in [3] using a modal logic. The subsequent sections on identity and unity don't make use of modal operators, but do use a temporal logic, in which each time-varying predicate has an extra temporal argument (e.g.  $P(a, t)$  indicates that  $P$  holds of  $a$  at time  $t$ ). A predicate of *actual existence* is also introduced. Among Guarino and Welty's papers to date, explication of the semantics of the existence predicate is limited to the following passage in [4]:

Our domain of quantification will be that of *possibilia*. That is, the extension of predicates will not be limited to what exists in the actual world, but to what exists in any possible world (Lewis, 1983). ... Actual existence is therefore different from existential quantification ("logical existence"), and will be represented by the temporally indexed predicate  $E(x, t)$ , meaning that  $x$  has actual existence at time  $t$  (Hirst, 1991).

where the Lewis reference is to [8], and the Hirst reference is to [6].

The mention of "what exists in the actual world" and "what exists in any possible world" in the explication of the time-indexed existence predicate would seem to indicate that the second argument of the existence predicate ranges over possible worlds, rather than times; and likewise, Guarino and Welty frequently (though not consistently) use terms like "essential" and "rigid" as if they involved quantification over time, rather than across worlds. For example, in [2, Section 4.2], "essential unity" is defined as something that holds of an individual if it is a contingent whole at every time where it exists; in [3, Section 5], the system asks the user: "Rigidity check: If an instance of animal ceases to be an instance of animal, does it cease to be an instance of physical-object?"; and in [3, Section 3.1], it is stated that the property *STUDENT* is not rigid because one can be a student for a time and then cease to be one.

There are precedents for equating times with possible worlds in Prior's temporal logic [10] and in Guarino's earlier work with Carrara and Giaretta [1]; the cited paper by Hirst also uses phrases like "the present real world" and "a past world." However, this reading is problematic for Guarino and Welty's logic because of the mixture of modal operators and explicit time arguments. In the existing literature, there are two

disjoint kinds of possible worlds logics: those with modal operators (which are interpreted as expressing implicit quantification over possible worlds), and those in which possible worlds are in the domain of quantification and can be referred to explicitly. Some logics of the first kind, *e.g.* that of Kripke [7], allow the definition of an existence predicate (via the axiom  $E(x) \leftrightarrow x = x$ ), but that predicate has no time/world index (since times/worlds are not denoted by terms of the logic). If we are to read [3] as presenting a single, unified theory, then we need to resolve how the modal operators and time arguments are related semantically.

The only natural way to interpret a logic that has both modal operators and explicit temporal arguments is to interpret the modal operators according to possible worlds semantics, take the existence predicate to be an ordinary predicate, and take the time arguments to be ordinary terms that range over the domain of quantification (which does not include possible worlds). Then worlds and times would be orthogonal—a proposition could be true in a world at one time, false in that world at another time, and false in another world at the same time. But as we have seen, this appears not to be Guarino and Welty’s intention—they don’t differentiate between times and worlds. Our only recourse seems to be to read the modal operators as quantifying implicitly over times, which are also in the domain of the quantifiers  $\exists$  and  $\forall$ . This is an entirely non-standard interpretation, so if it is what was intended, it needs to be stated explicitly.

While this seems to be the interpretation that brings the series of papers closest to consistency, it still leaves some questions. For example, a rigid property is defined as one for which

$$\forall x \phi(x) \rightarrow \Box \phi(x).$$

If we interpret  $\Box \phi(x)$  as meaning  $\forall t \phi(x, t)$ , then how should we translate the antecedent of the conditional? Ordinary modal logic uses a notion of the “current world,” but Guarino and Welty have not introduced the analogous “current time” in their hybrid modal/temporal logic. In an as yet unpublished paper [5], rigidity is redefined as

$$\phi(x, t) \rightarrow \Box \forall t' \phi(x, t').$$

This definition is similarly in conflict with a reading of the modal operator as quantifying implicitly over times, since it uses explicit quantification over times within the scope of a modal operator.

We are forced to conclude that Guarino and Welty’s temporal and modal “logic” simply has no consistent semantics. We have pointed out various ways in which the logic could be interpreted, but each of these choices requires the modification of some part of their framework.

#### 4. Identity

The concept of an *identity criterion* is introduced in [3, Section 3.2] by giving the formulation of [9]:  $\rho$  is an identity criterion for  $\phi$  if it is a relation of a suitable type (a qualification about which we will have more to say later) such that

$$\phi(x) \wedge \phi(y) \rightarrow (\rho(x, y) \leftrightarrow x = y). \quad (1)$$

Then, with the comment that “finding a  $\rho$  that is both necessary *and* sufficient for identity is often hard,” and introducing time arguments and the existence predicate

in order to differentiate between synchronic and diachronic identity, the notion of an identity criterion is separated into necessary and sufficient identity conditions: formula  $\Gamma$  is a necessary identity condition for  $\phi$  (also phrased as “ $\phi$  carries identity condition  $\Gamma$ ”) if

$$E(x, t) \wedge \phi(x, t) \wedge E(y, t') \wedge \phi(y, t') \wedge x = y \rightarrow \Gamma(x, y, t, t') \quad (2)$$

and is a sufficient identity condition if

$$E(x, t) \wedge \phi(x, t) \wedge E(y, t') \wedge \phi(y, t') \wedge \Gamma(x, y, t, t') \rightarrow x = y \quad (3)$$

These definitions are stated in [3] with the qualification “excluding trivial cases.” In [4], the qualification is specified in logical form: both necessary and sufficient conditions are constrained by

$$\neg \forall xy \Gamma(x, y, t, t') \leftrightarrow x = y \quad (4)$$

and necessary conditions are subject to the constraint

$$\neg \forall xy E(x, t) \wedge \phi(x, t) \wedge E(y, t') \wedge \phi(y, t') \rightarrow \Gamma(x, y, t, t') \quad (5)$$

while sufficient conditions are subject to the constraint

$$\exists xytt' \Gamma(x, y, t, t') \quad (6)$$

If a property  $\phi$  has some (necessary or sufficient) identity condition, Guarino and Welty write  $\phi^{I+}$ ; if it has no identity condition, they write  $\phi^{I-}$ . Properties classified **+I** are called “sortals,” following Strawson [11].

We will now show that the features **+/-I** do not, in fact, make a useful discrimination between different kinds of properties. Having done so, we will examine the intuitions that were apparently behind Guarino and Welty’s definitions, and propose an alternate formulation that more correctly expresses these intuitions.

#### 4.1 Failure of the Non-Triviality Constraints

The **+/-I** classification is meaningless given the definitions Guarino and Welty use, because under these definitions the only properties that *don’t* have identity conditions are ones that belong to a particular class of trivial properties, as we will now see. First, consider the situation if we omitted the non-triviality constraints from the definitions of identity conditions:

**THEOREM 5.** *If constraints (4), (5), and (6) are omitted from the definitions of necessary and sufficient identity conditions, then there is an identity condition that every property carries.*

**PROOF.** If the non-triviality constraints are left out of the definition, then the identity relation itself is a necessary and sufficient identity condition for every property: substituting  $x = y$  for  $\Gamma(x, y, t, t')$  in (2) and (3) results in a tautology, regardless of what property  $\phi$  is.  $\square$

Similarly, if we consider merely necessary or merely sufficient conditions, any tautologically true formula would be a necessary identity condition, and any tautologically false formula would be a sufficient identity condition, for all  $\phi$ . Guarino and Welty appear to have noticed these problems, because they have added constraints (4), (5), and

(6) to disallow these trivial conditions explicitly. (5) also rules out the slightly less trivial formula  $E(x, t) \wedge \phi(x, t) \wedge E(y, t') \wedge \phi(y, t')$  as a necessary identity condition. They apparently judged, quite justifiably, that these trivial formulae are not useful or informative identity conditions; but ruling out these specific relations was the wrong solution. These aren't the only trivial or uninformative identity conditions, they are simply three of the most natural ones. There are an infinity of other equally uninteresting identity criteria that the purported non-triviality constraints don't rule out. The following theorem gives one example.

**THEOREM 6.** *There is a sufficient identity condition that is carried by every property that has two or more instances.*

**PROOF.** Consider the formula  $x = y \wedge x \neq c$ , for some arbitrarily chosen  $c$ . Substituting this formula for  $\Gamma(x, y, t, t')$  makes (3) and (4) tautologies. It doesn't make (6) a tautology, but it makes it true in every model that satisfies  $\exists x x \neq c$ , *i.e.* models whose domain of interpretation contains two or more elements. Therefore,  $x = y \wedge x \neq c$  is a sufficient identity condition for every property that holds of at least two things.  $\square$

The constructed identity condition could be modified to yield a condition carried by even more properties, and similar constructions can be invented to circumvent the *ad hoc* constraints placed on necessary identity conditions; but even without going through such an exercise, we have already demonstrated that only a very limited class of properties are to be labeled **-I** according to Guarino and Welty's definitions, and therefore the meta-property **+/-I** cannot play the major role they intend for it in structuring taxonomies.

It should be clear by now that the difference between interesting and non-interesting identity conditions is not to be found in their logical properties. The statement "two lumps of clay are identical if they have the same parts" is more informative than "two lumps of clay are identical if they are identical" because the former reduces the question of identity to a different question about which we might have clearer intuitions. "Two lumps of clay are identical if they are identical" also happens to be tautological, but that is not the relevant problem.

## 4.2 Identity and Subsumption

Constraint (7) in [3] states that if  $\phi$  carries some identity condition and  $\psi$  carries no identity condition, then  $\phi$  can't subsume  $\psi$ . Given what we have just seen, this is not a useful constraint for structuring taxonomies of non-trivial properties. However, this is not to say that the notion of identity conditions has nothing to contribute to understanding taxonomic structure. Constraint (11) of [3] says that "properties with incompatible identity conditions are disjoint." This constraint refers to the specific identity conditions that a property carries, rather than simply to the question of whether or not it carries any, and therefore escapes triviality. As we will now see, the constraint is ill-defined, but it is on the right track.

In [2], Guarino and Welty take the following quote from Lowe [9] as a starting point:

No individual can instantiate both of two sorts if they have different criteria of identity associated with them.

It is then pointed out that Lowe's principle is insufficiently precise: "having the same edges and the same angles" is an identity criterion for polygons (including triangles),

while “having two edges and their internal angle in common” is an identity criterion for triangles but not for polygons. The class of polygons and that of triangles carry different identity conditions, but they are not disjoint; in fact, the former subsumes the latter.

This point is well taken. Guarino and Welty’s alternative principle is that if two properties have “incompatible” identity conditions, then they are disjoint. Unfortunately, the term “incompatible” is not defined, and the example given sheds no light on what it is intended to mean. The example is that the classes “amount of matter” and “person” must be disjoint because replacing some of the parts of an amount of matter makes it a different amount of matter, while a person can remain the same person while some of his parts are replaced (the identity condition being implicitly referred to is presumably the binary relation “are composed of the same parts”). But in the relevant aspects, this example is no different from the polygon/triangle example: it is simply another instance of one class carrying an identity condition that the other doesn’t. Perhaps an apparent difference is that polygon and triangle, in addition to the identity condition on which they differ, also share an identity condition. But for the amount of matter/person example to be used as a contrast to this, it would have to be demonstrated that there is no identity condition that the two properties share. Guarino and Welty do not show this, and indeed it is difficult to imagine how it could be shown.

Although there doesn’t appear to be a useful interpretation of the  $+/-\mathbf{I}$  notation, there is a constraint on taxonomic relationships that can be stated in terms of identity criteria, and even merely necessary or merely sufficient identity conditions. Quite simply,

**THEOREM 7.** *If  $\phi$  carries identity condition  $\Gamma$ , and  $\phi$  subsumes  $\psi$ , then  $\psi$  carries  $\Gamma$  as well; or, stated another way, if  $\phi$  carries identity condition  $\Gamma$  and  $\psi$  does not, then  $\phi$  does not subsume  $\psi$ .*

The proof is trivial. Note that this constraint generates the correct results for both of the above examples. Triangle carries an identity condition that polygon does not, so triangle doesn’t subsume polygon; in the other direction, the example doesn’t mention any identity condition that polygon carries but triangle does not, so the constraint doesn’t apply. Likewise, “amount of matter” carries an identity condition that “person” does not, so “amount of matter” doesn’t subsume “person.”

## 5. Unity

Where  $P(x, y, t)$  means that  $x$  is a part of  $y$  at time  $t$  (the predicate  $P$  is constrained axiomatically), an object  $x$  is said to be a *contingent whole* under the *unifying relation*  $\omega$  at time  $t$  if

$$\forall y (P(y, x, t) \rightarrow \forall z (P(z, x, t) \leftrightarrow \omega(z, y, t))) \quad [2, \text{formula (16)}] \quad (7)$$

$$\neg \forall xyz t P(y, x, t) \wedge P(z, x, t) \leftrightarrow \omega(y, z, t) \quad [2, \text{formula (17)}]^2. \quad (8)$$

(7) gives the core meaning of the term “unifying relation:” it says that  $\omega$  is a unifying relation for  $x$  if all of the parts of  $x$  are related to each other by the relation  $\omega$ .

<sup>2</sup>[2, formula (17)] contained a typographical error. The formula presented here is the intended axiom, as confirmed by Guarino in personal communication.

(8) is said to be a non-triviality condition, which immediately raises suspicions given the problems with the so-called non-triviality conditions for identity discussed in Section 4.1. In fact, the problems with this formula are manifold: there are problems both with the concept underlying the constraint, and with the attempt to express that concept in logical form.

From the text that follows the formulas, it seems that the intended constraint is that there must be some values of  $x$  and  $t$  for which (7) doesn't hold, *i.e.* that  $\omega$  must not be the universal unifying relation. First of all, this constraint is just *ad hoc*, and therefore just as ineffective, as those discussed in Section 4 above. It rules out a single undesirable relation (the universal unifying relation) while allowing an infinity of other undesirable relations that are just minor modifications of it.

Furthermore, (8) doesn't even express the apparently intended constraint. If the intention is indeed that there are some  $x$  and  $t$  for which (7) doesn't hold, the logical expression of that condition would be

$$\neg \forall xyt (P(y, x, t) \rightarrow \forall z (P(z, x, t) \leftrightarrow \omega(z, y, t))), \quad (9)$$

which is not logically equivalent to (8).

The notion of *intrinsic whole* is defined as follows:

An object  $x$  is an *intrinsic whole under*  $\omega$  if, at any time where  $x$  exists, it is a contingent whole under  $\omega$  [2, Definition 6].

This definition is not stated in logical form, but since (7) gives the logical definition of a contingent whole, the logical definition of “ $x$  is an intrinsic whole under  $\omega$ ” must be

$$\forall t E(x, t) \rightarrow \forall y (P(y, x, t) \rightarrow \forall z (P(z, x, t) \leftrightarrow \omega(z, y, t))).$$

Simplifying, and using the axiom  $P(x, y, t) \rightarrow E(x, t) \wedge E(y, t)$  given in [2, Table 1], this definition reduces to

$$\forall tyz P(y, x, t) \rightarrow (P(z, x, t) \leftrightarrow \omega(z, y, t)). \quad (10)$$

Next, the meta-property of *carrying a unity condition* is defined:

A property  $\phi$  *carries a unity condition* if there is a relation  $\omega$  such that instances of  $\phi$  are intrinsic wholes under  $\omega$  [2, Definition 7].

The fact that property  $\phi$  carries a unity condition is expressed with the notation  $\phi^{+U}$ . Building on (10), the logical form of this definition is that  $\phi$  carries a unity condition if there is a relation  $\omega$  such that

$$\forall xtyz \phi(x, t) \wedge P(y, x, t) \rightarrow (P(z, x, t) \leftrightarrow \omega(z, y, t)). \quad (11)$$

We will now see that, like the *+/-I* distinction, the *+/-U* distinction is not as useful a tool for structuring taxonomies as Guarino and Welty intended it to be. In particular, they claim that the properties whose instances are countable are those that are both *+I* and *+U* [2, Section 5]; since we have already shown that only trivial properties are not *+I*, the claim reduces to the claim that the meta-property *+/-U* is what differentiates between countable and uncountable properties. This is not the case, as we will now see.

Let us define *properly overlapping parts* as follows: two entities  $A$  and  $B$  have properly overlapping parts if there is something which is a part of both  $A$  and  $B$ , and

there is something else which is a part of  $A$  but not  $B$ , or vice versa. Theorem 8 below states that the properties that carry unity conditions are merely those whose instances can have properly overlapping parts. ‘Committee’ is an example of a property whose instances can have properly overlapping parts: it is possible for committee  $A$  to consist of persons  $a$  and  $b$ , and committee  $B$  to consist of persons  $a$  and  $d$ . Person  $a$  is on both committees, but  $b$  and  $d$  are on only one committee each.

**THEOREM 8.** *Property  $\phi$  carries a unity condition if and only if for any two instances  $A$  and  $B$  of  $\phi$ , either no part of  $A$  is part of  $B$  and vice versa, or every part of  $A$  is part of  $B$  and vice versa.*

**PROOF.** We will show first the “only if” direction, and then the “if” direction.

Assume, for the purpose of deriving a contradiction, that  $\phi$  carries unity condition  $\omega$ , as defined by (11), and that the following assertions are true:  $\phi(A, t_1)$ ,  $\phi(B, t_1)$ ,  $P(a, A, t_1)$ ,  $P(a, B, t_1)$ ,  $P(b, A, t_1)$ ,  $\neg P(b, B, t_1)$ . By instantiating (11) with  $A$  for  $x$ ,  $t_1$  for  $t$ ,  $a$  for  $y$ , and  $b$  for  $z$ , it follows that  $\omega(b, a, t_1)$  is true. Then, instantiating it again as before except with  $B$  for  $x$ , it follows that  $P(b, B, t_1)$ , contradicting our initial assumptions.

For the “if” direction, consider the unifying relation defined by

$$\forall zyt \omega(z, y, t) \leftrightarrow \exists x \phi(x, t) \wedge P(z, x, t) \wedge P(y, x, t). \quad (12)$$

Substituting the right-hand side of this definition for  $\omega(z, y, t)$  in (11) results in

$$\forall xtyz \phi(x, t) \wedge P(y, x, t) \rightarrow (P(z, x, t) \leftrightarrow \exists x \phi(x, t) \wedge P(z, x, t) \wedge P(y, x, t))$$

which is equivalent to the conjunction of

$$\forall xtyz \phi(x, t) \wedge P(y, x, t) \wedge P(z, x, t) \rightarrow \exists x \phi(x, t) \wedge P(z, x, t) \wedge P(y, x, t),$$

which is a tautology, and

$$\forall xtyz \phi(x, t) \wedge P(y, x, t) \wedge \exists x \phi(x, t) \wedge P(z, x, t) \wedge P(y, x, t) \rightarrow P(z, x, t).$$

The latter is not a tautology, but it does follow from

$$\forall xyzwt P(y, x, t) \wedge P(z, w, t) \wedge P(y, w, t) \rightarrow P(z, x, t)$$

which is precisely the constraint that if two wholes share any parts, then they share all of their parts.  $\square$

In other words, the distinction +/-U differentiates between properties whose instances can have properly overlapping parts, and those that don’t. As the ‘committee’ example shows, there is no justification for identifying the countable properties with those that carry unity conditions (the property is countable, but doesn’t carry a unity condition).

In [4], Guarino and Welty also define a label  $\sim U$ , for anti-unity, as follows: a property has anti-unity if every instance of the property is not an intrinsic whole. We will show that this class is in fact empty: for every object, *i.e.* every instance of every property, there is a relation under which the individual is an intrinsic whole. While this relation is not always non-trivial according to constraint (8) or its amended version (9), we will show that one innocuous stipulation guarantees its non-triviality.

**THEOREM 9.** *For every object  $a$ , i.e. every instance of every property, there is a relation under which the individual is an intrinsic whole. Under the stipulation  $\exists xt \neg P(x, a, t)$ , i.e. the stipulation that there is something that is not always part of  $a$ , that relation satisfies (8) and (9).*

**PROOF.** Any object  $a$  is an intrinsic whole under the relation  $\omega$  defined by

$$\forall yzt \omega(y, z, t) \leftrightarrow P(y, a, t) \wedge P(z, a, t).$$

Substituting the right-hand side of this biconditional for  $\omega(y, z, t)$  in (8) yields

$$\neg \forall xyz t (P(y, x, t) \wedge P(z, x, t)) \leftrightarrow (P(y, a, t) \wedge P(z, a, t)),$$

which is a logical consequence of  $\exists xt \neg P(x, a, t)$ . In other words, the relation  $\omega$  is non-trivial whenever  $\exists xt \neg P(x, a, t)$  holds. A parallel argument leads to the same result for the amended non-triviality condition (9).  $\square$

As with identity conditions, the true usefulness of unity conditions in constraining taxonomic relations lies not in determining which properties carry unity conditions and which don't (if we accept Guarino and Welty's definition of "carries a unity condition"), but in choosing particular interesting unity conditions, and determining which properties carry *them*. Like identity conditions, unity conditions are inherited from subsuming properties, so if  $\phi$  carries unity condition  $\omega$  and  $\psi$  doesn't, then  $\phi$  does not subsume  $\psi$ .

## 6. Conclusions

Guarino and Welty's work on formalizing taxonomic constraints is based on some useful intuitions, but the mathematical execution is flawed. This paper is an attempt to repair a number of problems. The major results can be summarized as follows.

(1) Guarino and Welty's notation and associated descriptions indicate a confusion between modal logic on the one hand, and temporal logic with explicit temporal arguments on the other, with the result that their formulae are uninterpretable by any standard semantics. The way to alleviate this confusion with the least disturbance to their framework would seem to be to replace all modal operators with explicit quantification over times, and to state explicitly that terms like "necessary," "rigid," and "essential" are related to persistence over time, rather than across worlds.

(2) The claim that an anti-rigid property can't subsume a rigid one is false. Incorrect intuitions about this issue probably indicate a confusion between subsumption and necessary subsumption, since it is true that an anti-rigid property can't subsume a rigid one *necessarily*.

(3) The meta-properties  $+/-\mathbf{I}$  and  $+/-\sim\mathbf{U}$ , indicating whether or not a property carries identity or unity conditions, turn out to be useless for constraining taxonomic relationships, as they are defined by Guarino and Welty. However, the property of carrying a *particular* identity or unity condition does place useful constraints on taxonomic relations between properties.

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