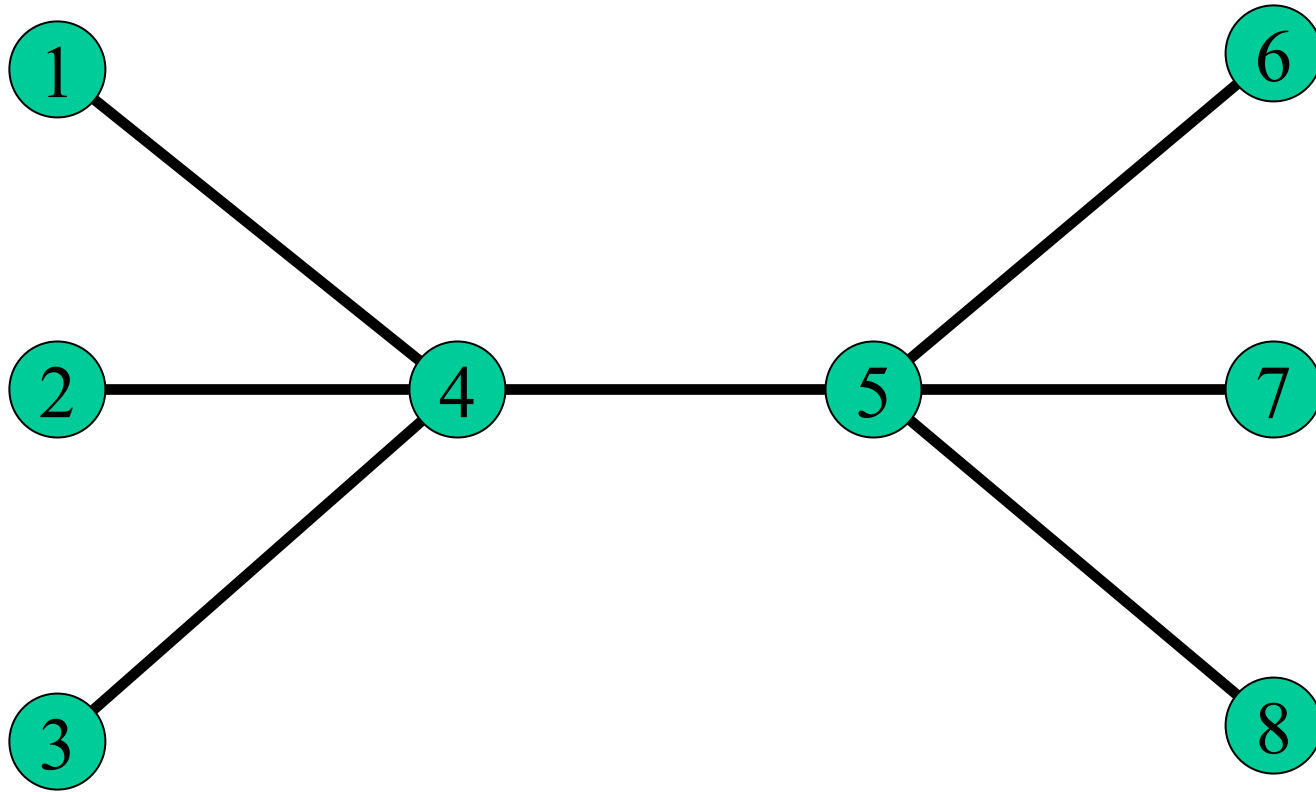


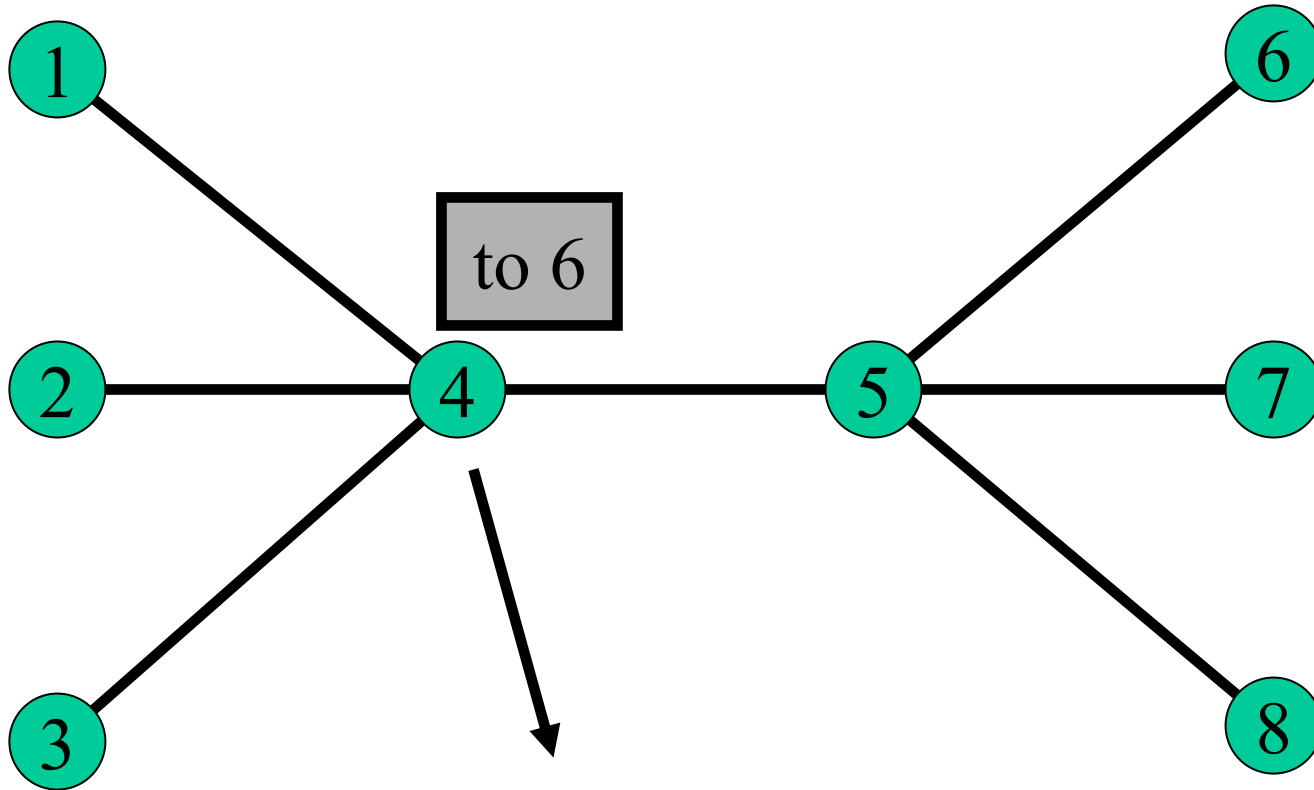
Acyclic orientations do not lead to optimal deadlock-free packet routing algorithms.

Daniel Štefankovič

# Packet routing

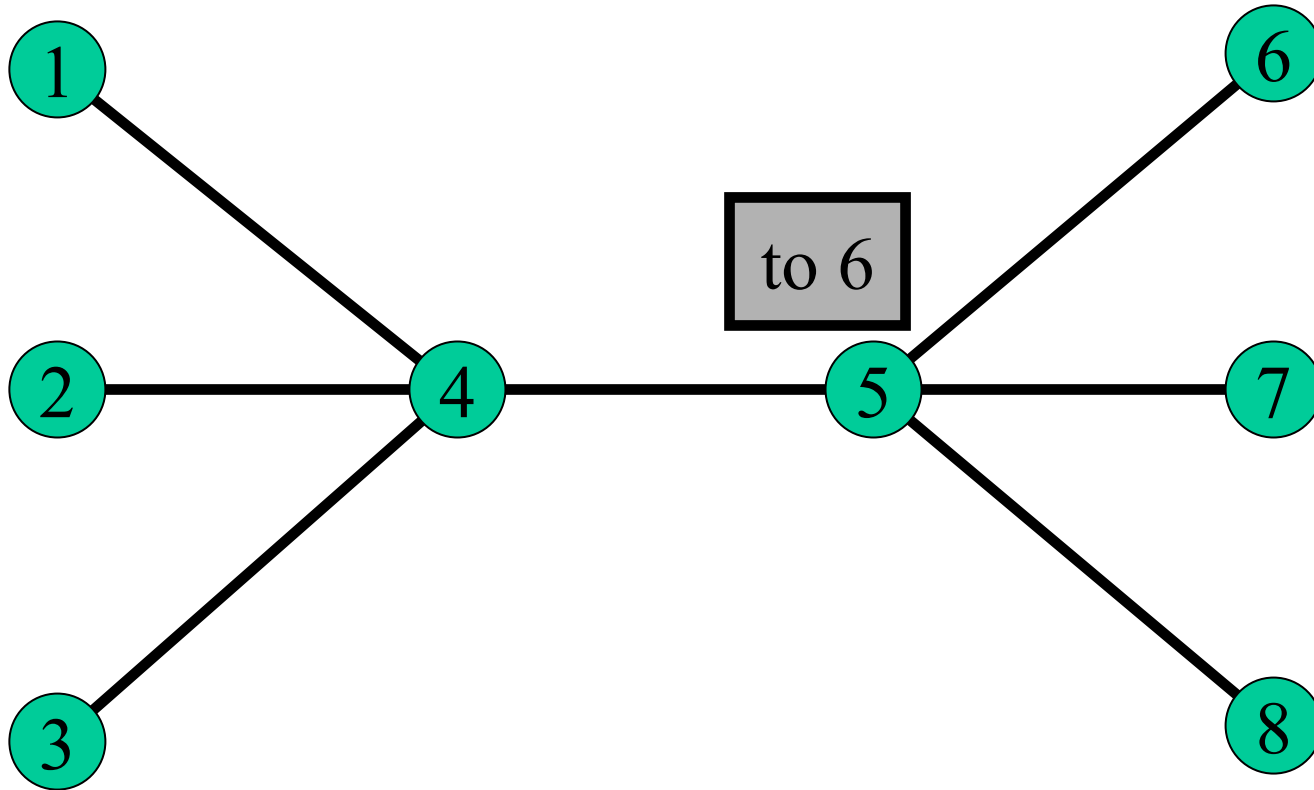


# Routing – one packet

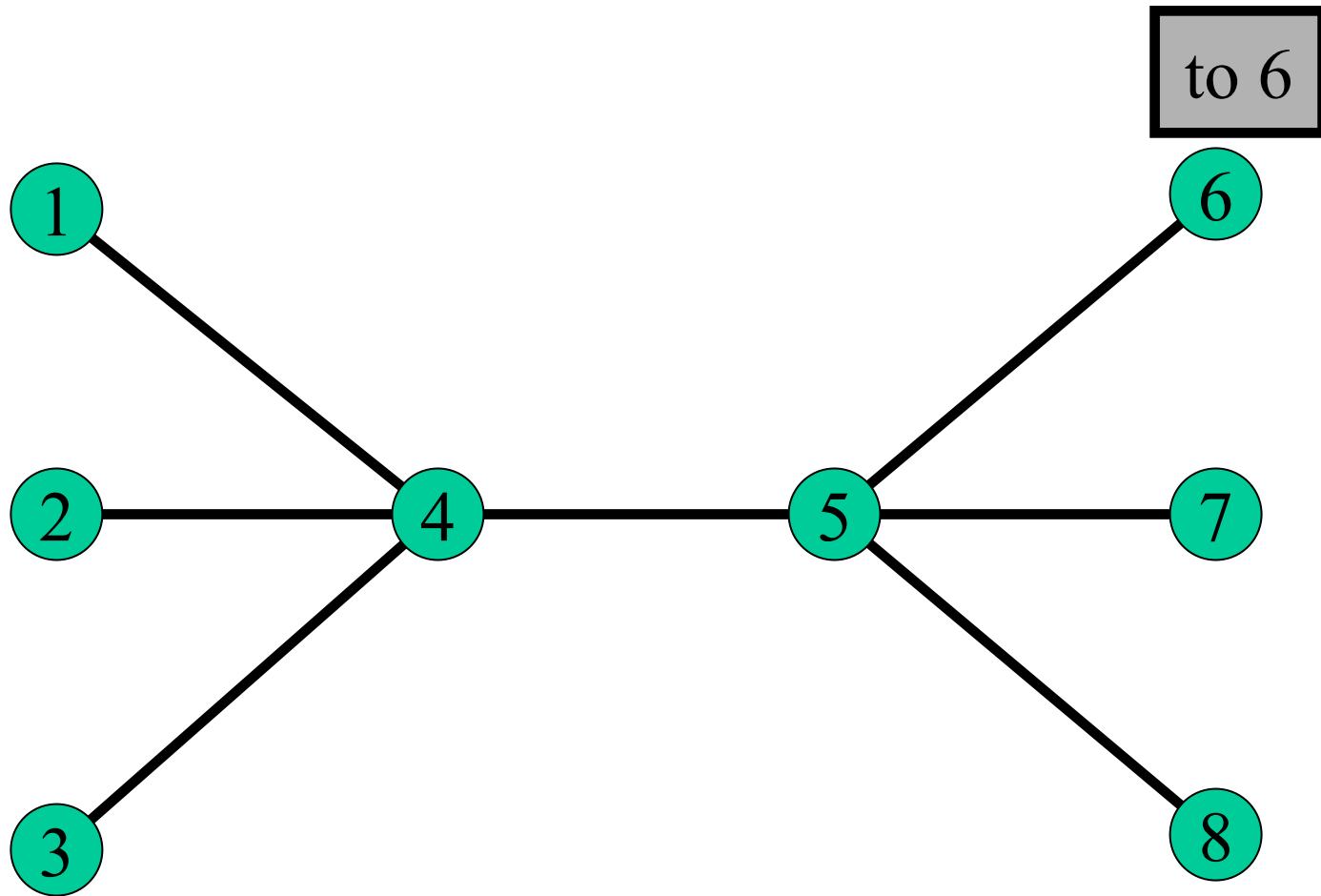


Where should I forward the packet?

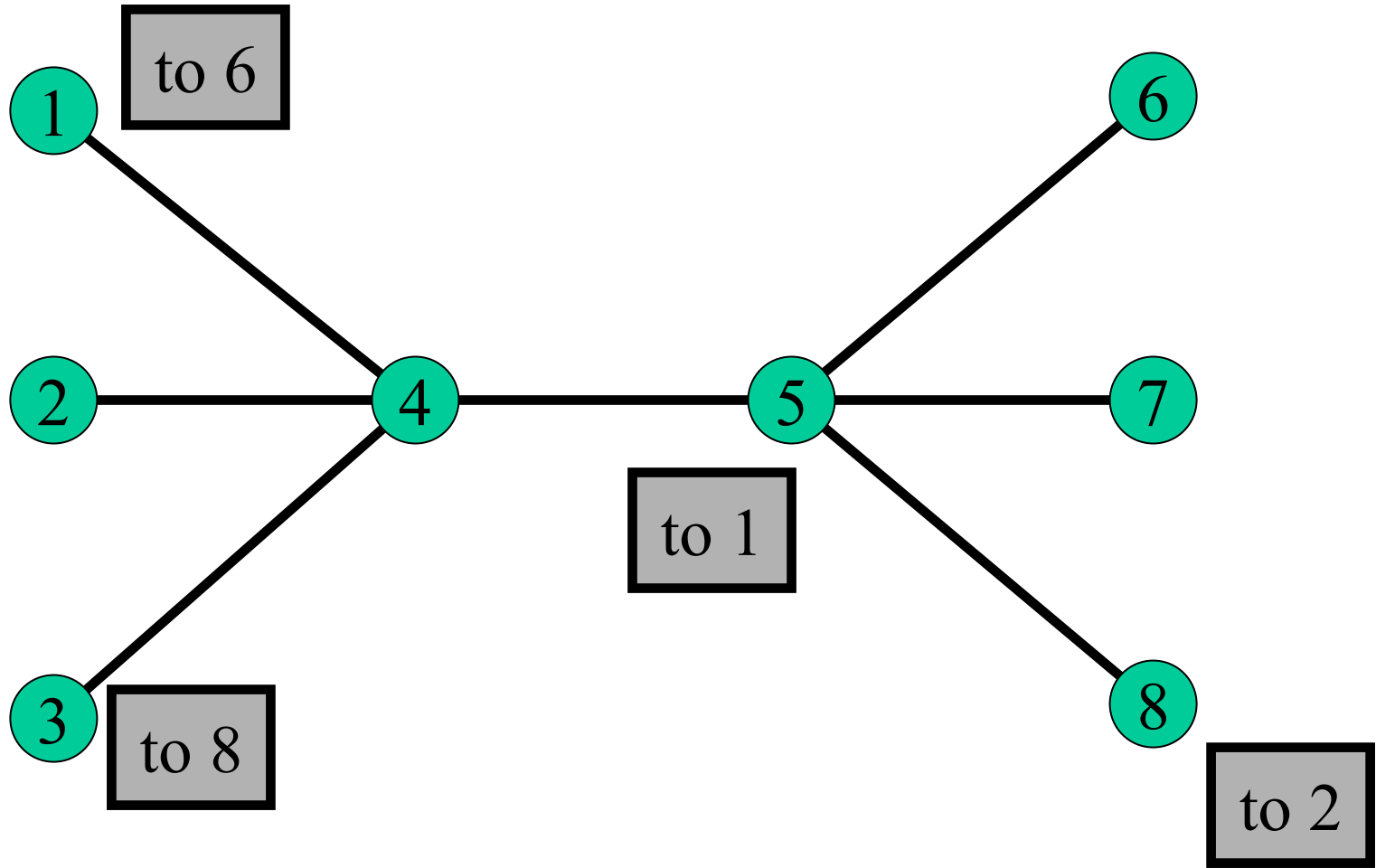
# Routing – one packet



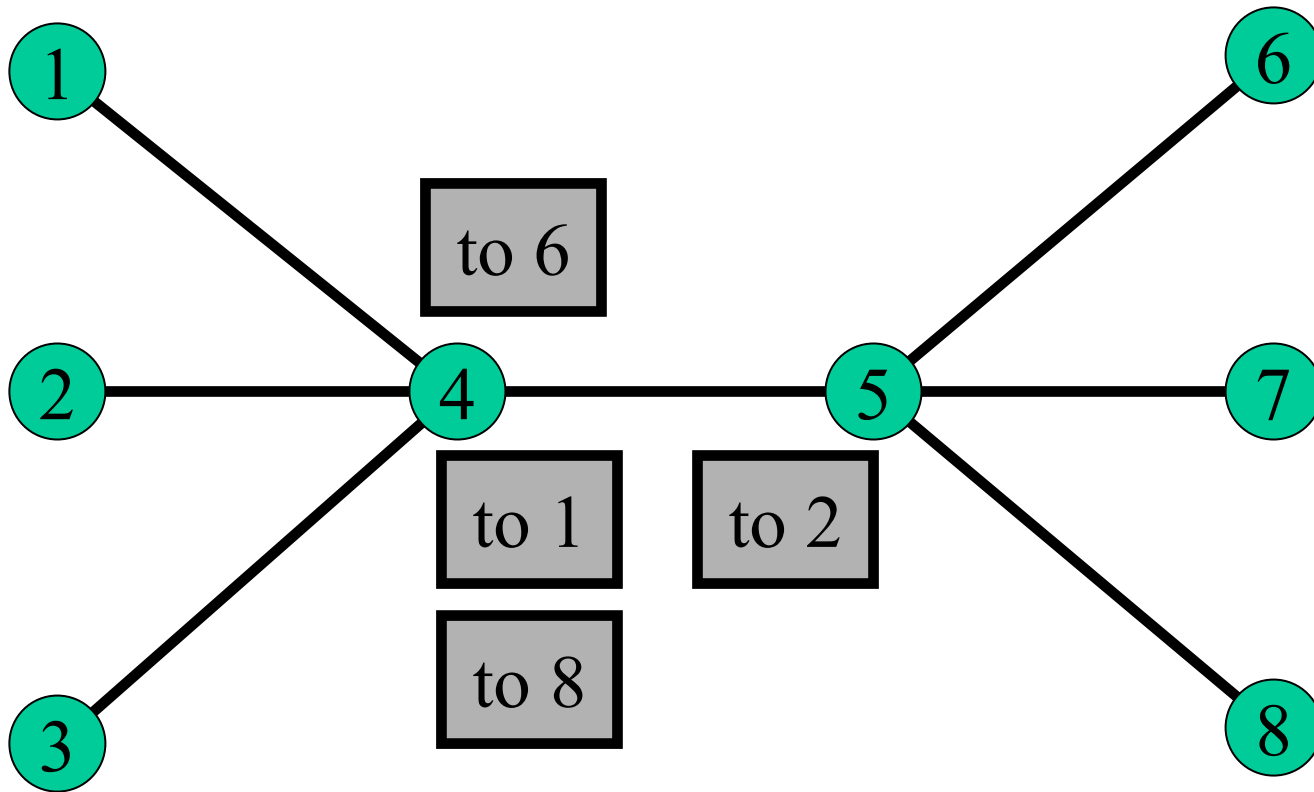
# Routing – one packet



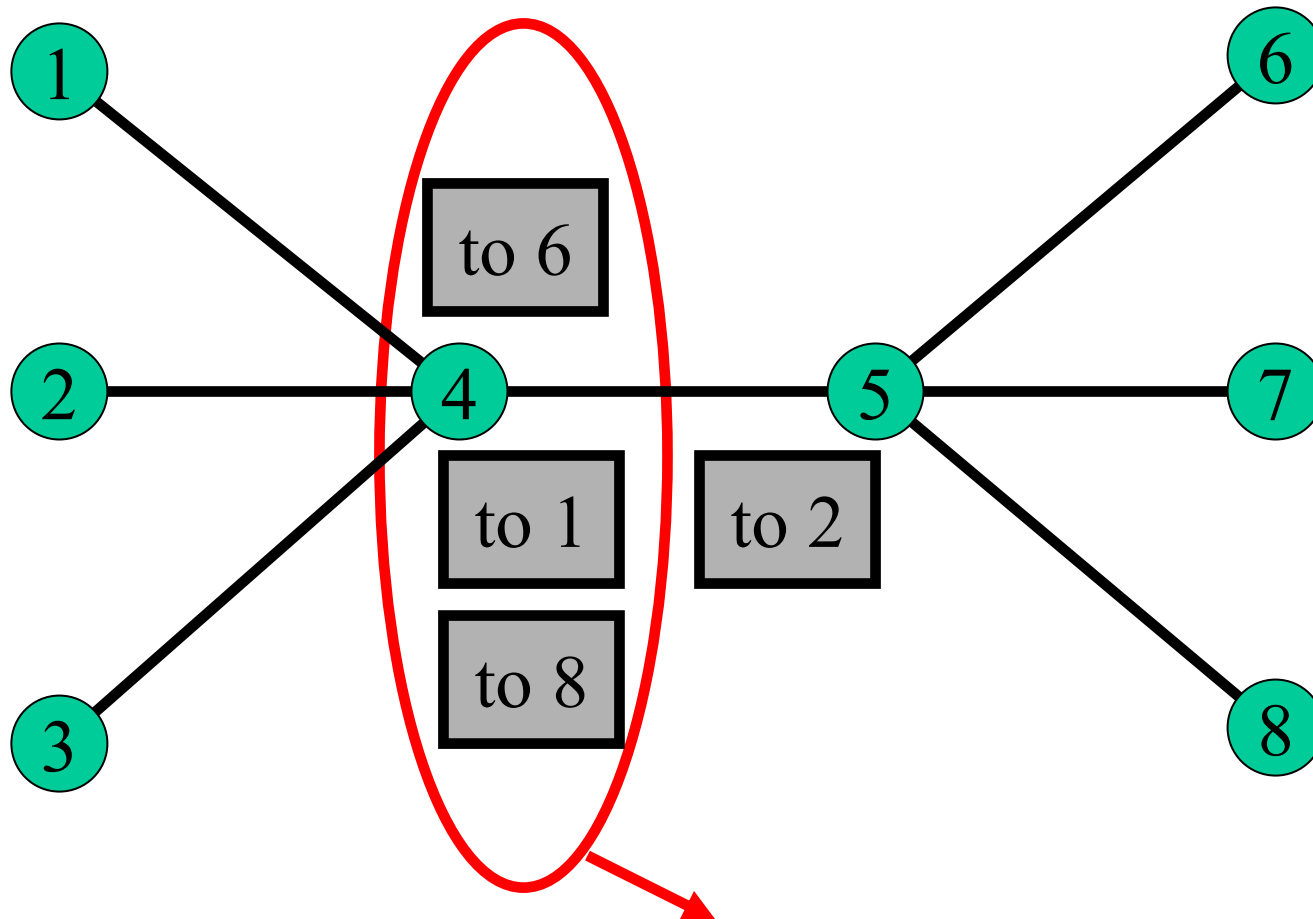
# Routing – more packets



# Routing – more packets



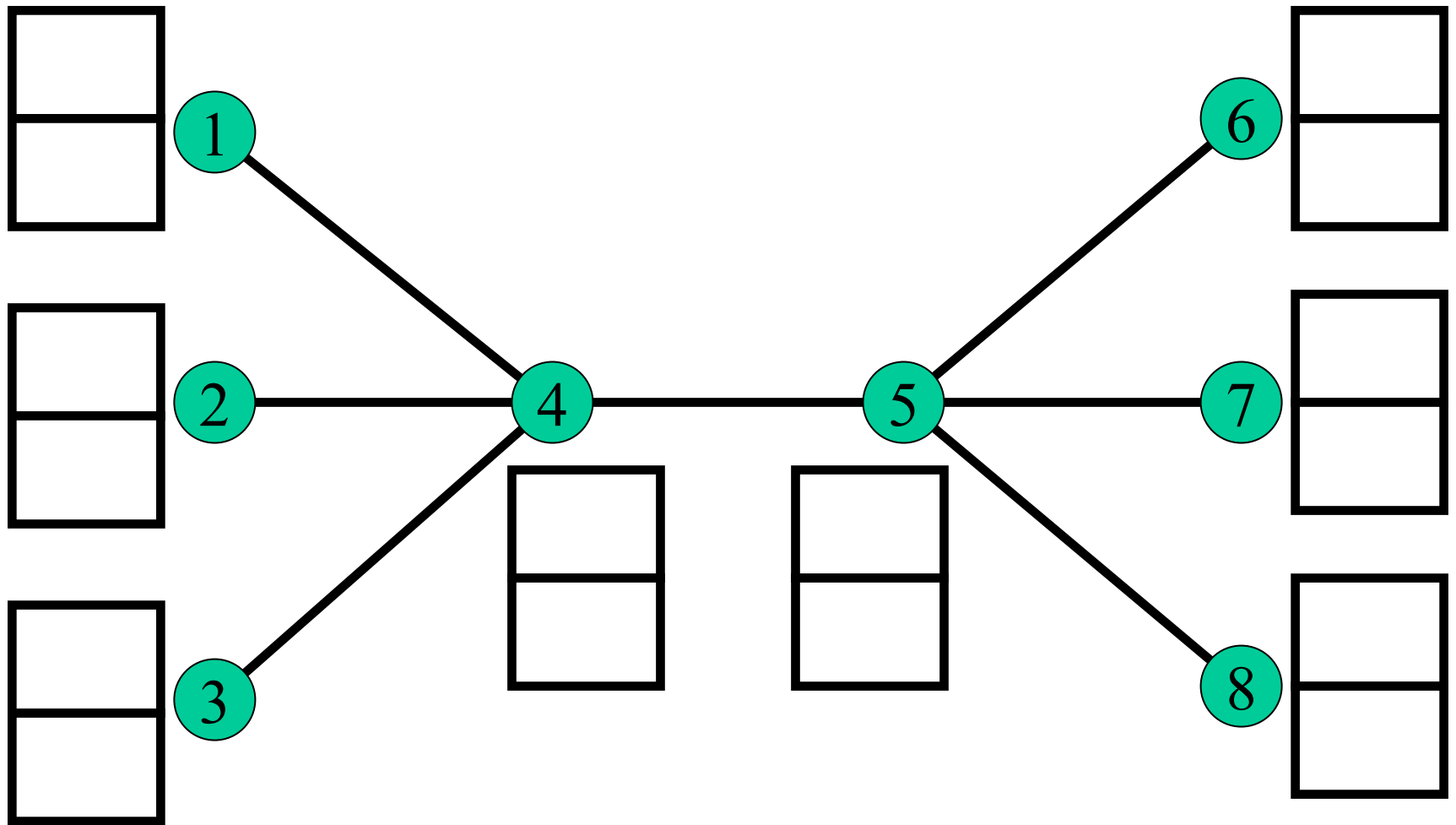
# Routing – more packets



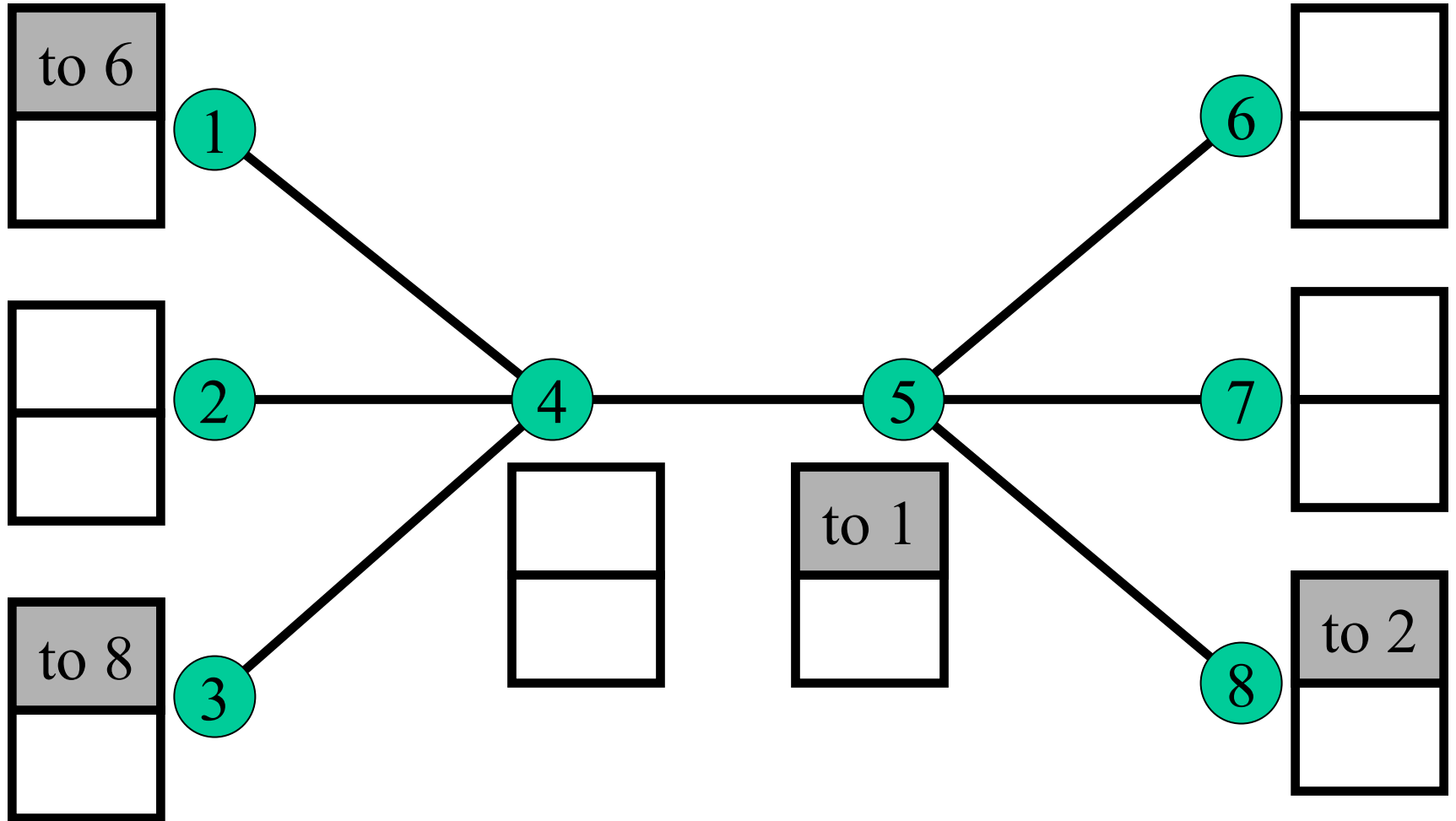
We need to store the packets



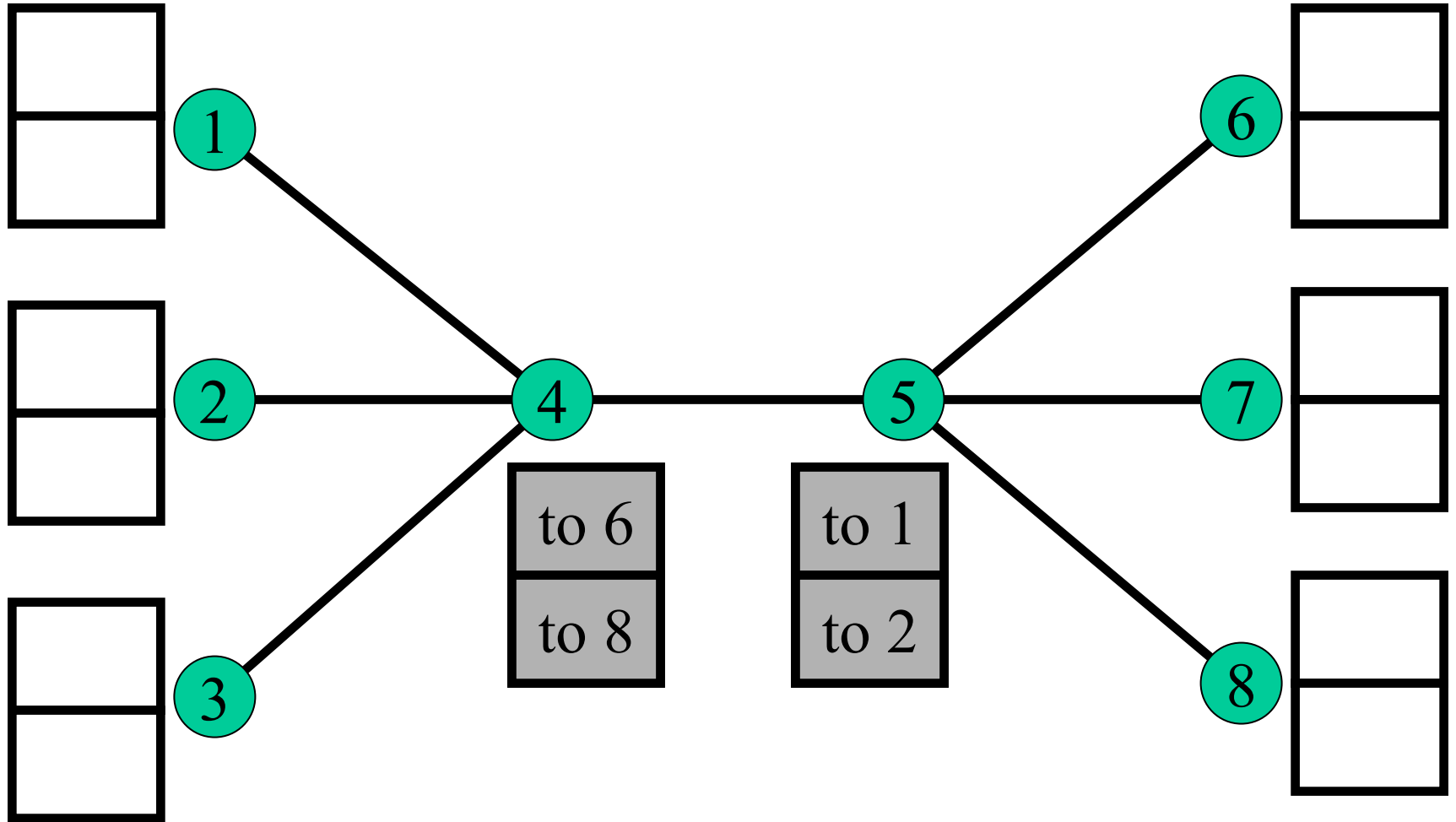
# Buffers



# Routing with buffers



# Store&forward deadlock



# The model (buffer reservation)

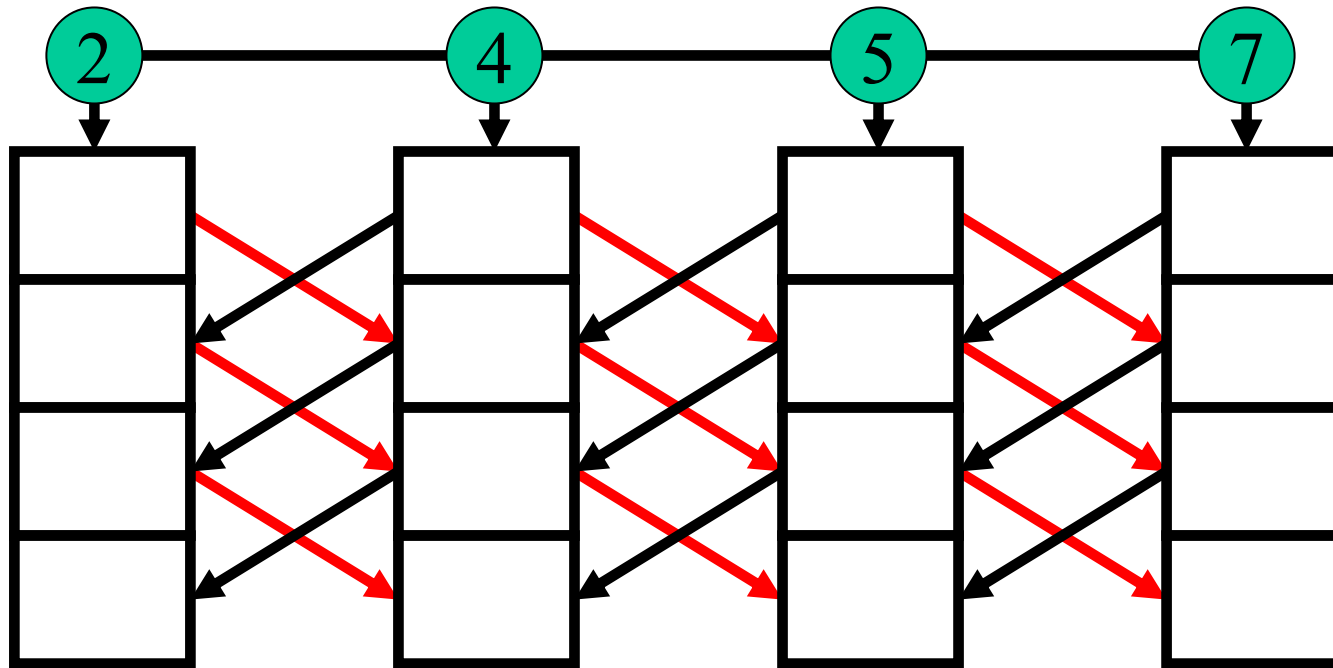
- the packets must go along some shortest route
- the devil controls the race conditions
- we control to which buffer the packet goes



Can we avoid store&forward deadlock?

# Yes, we can

we control to which buffer the packet goes



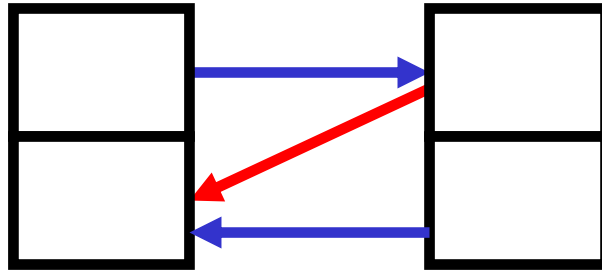
# How many buffers per node suffice?

$\text{diam}(G)+1$  buffers are enough

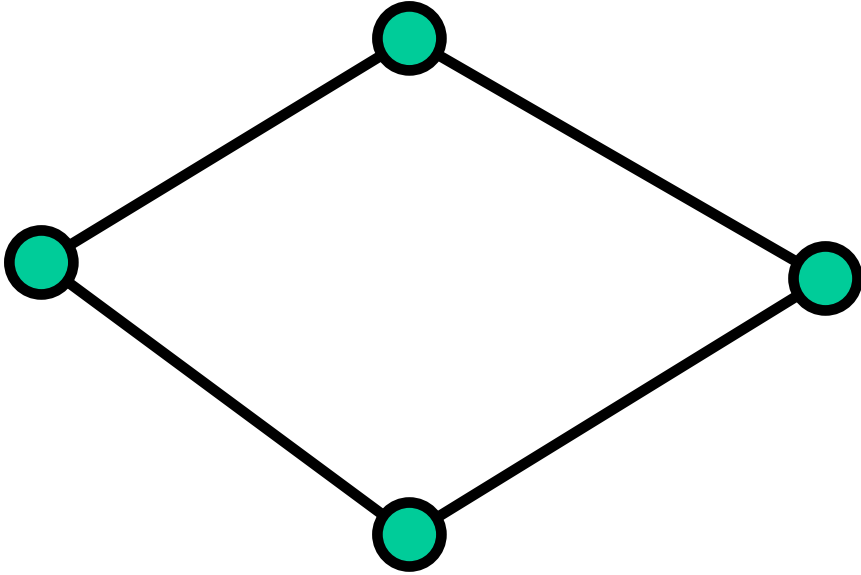
(always move from buffer  $i$  to buffer  $i+1$ )

Can we do better?

# Can we do better?

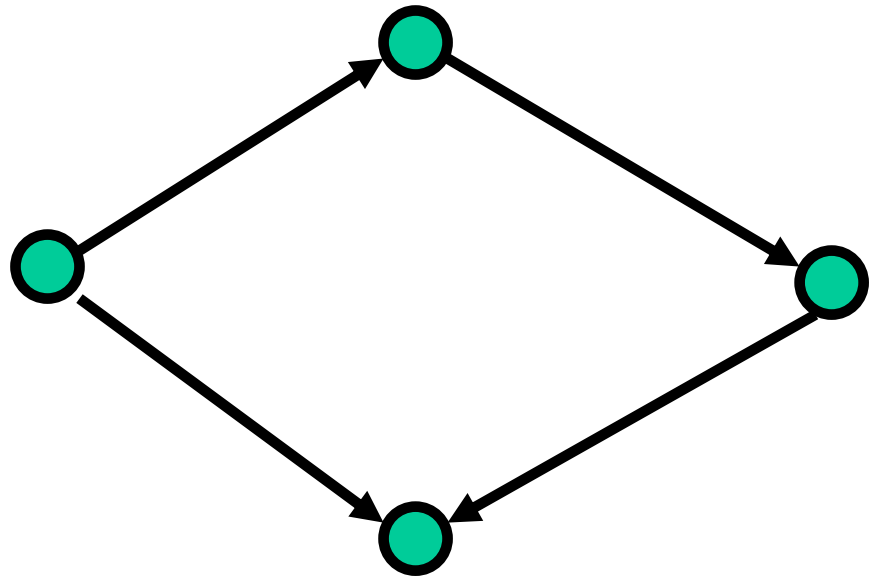
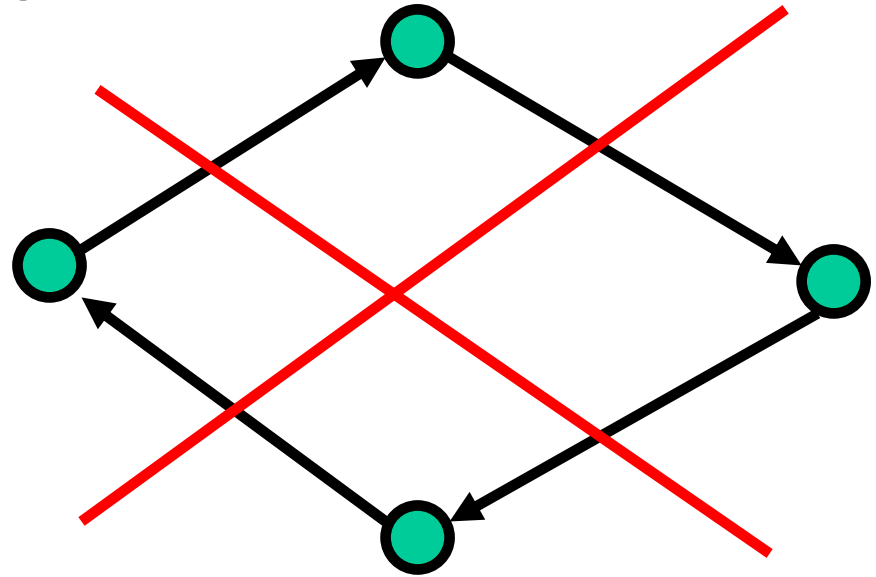
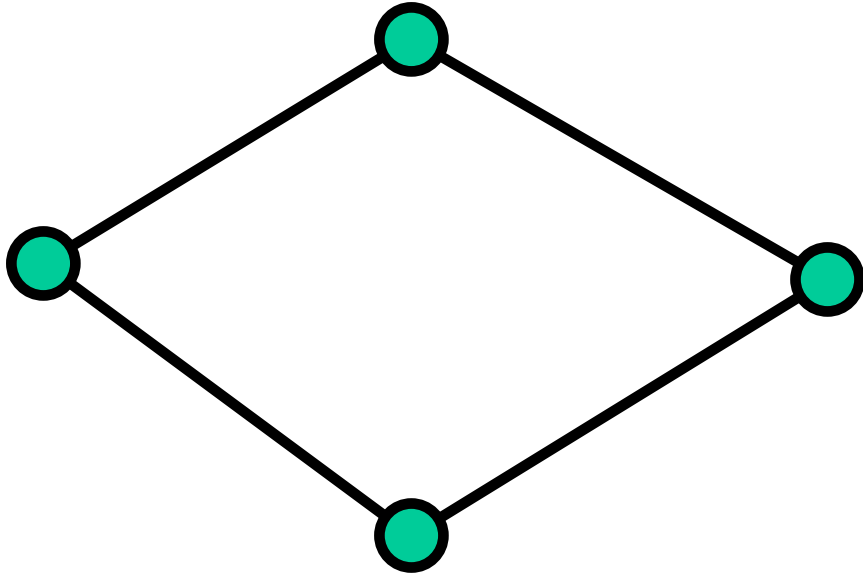


# Acyclic orientations





# Acyclic orientations



# Acyclic orientations

If all packets are travelling along acyclic orientation then there cannot be deadlock.



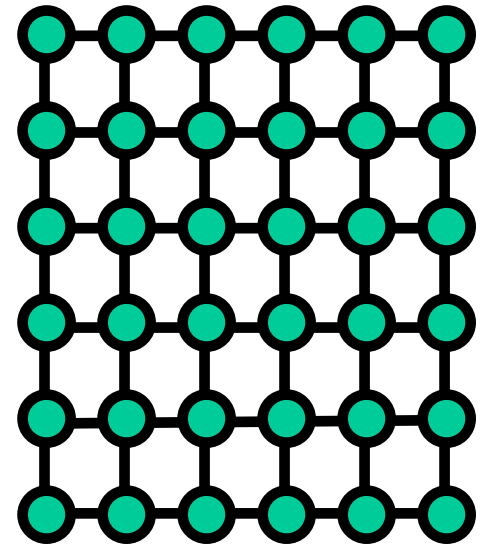
Split routing into phases, in each phase packets would travel along an acyclic orientation.

# Orientation cover of a set of paths $P$

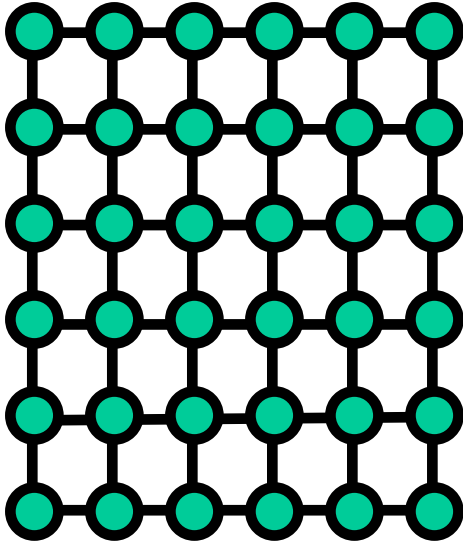
Set of acyclic orientations  $A_1 A_2 \dots A_k$  such that each shortest path  $p \in P$  can

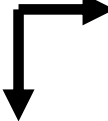
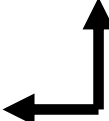
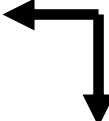
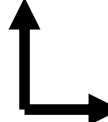
be written as  $p_1 p_2 \dots p_k$  where  $p_i$  is the longest prefix of  $p_i \dots p_k$  which can be traversed in  $A_i$ .

$P$  – for any two vertices  $u, v$  at least one shortest path between  $u, v$



# Orientation cover of a grid



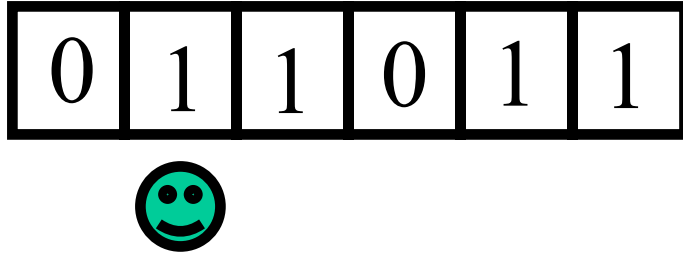
1. 
2. 
3. 
4. 

# The question:

Is it always possible to design (asymptotically) optimal solution using acyclic orientations?

There is a graph  $G$  with  $N$  vertices which has buffer reservation scheme with 8 buffers per vertex but any orientation cover of  $G$  has size  $\Omega(\log N / \log \log N)$ .

# The graph G



Machine M consists of a tape with  $n$  cells and a head which can be positioned above any cell. In one step head can either change the content of the occupied cell or move to the left or to the right.

# Shortest paths in G



# Solution with 8 buffers



1 ←

→ 2

3 ←



→ 4

3 ←

→ 2

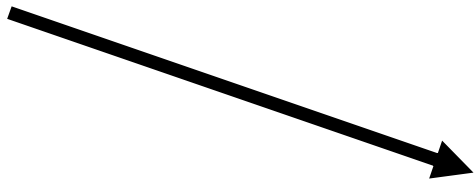
shuffle aa'



# The lower bound

1. Concentrate only on the orientation of the shuffle edges, the other edges are free.

2. Consider only paths from vertices having head on the leftmost cell to vertices having head on the rightmost cell.



unique shortest paths

# The lower bound

A path in hypercube  $Q_n$  is **monotone** if the bits are changed from left to right.

What is the minimal size of a cover of the set of all monotone paths in the hypercube  $Q_n$ ?

# The lower bound

Everybody sends packet to everybody.

We are going to observe the system  
after each orientation.

# d-active vertex



If for each vertex  $u$  of the form  $u=xz$  there is a packet in the system destined to  $u$  which has to pass through  $v$ .

(i.e. it was sent by somebody of the form  $wy$ .)

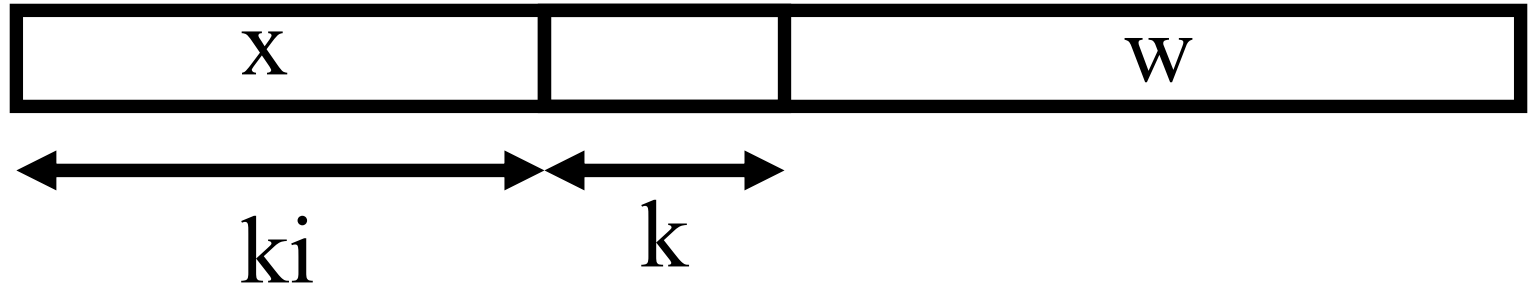
If  $u$  has not yet received a packet from somebody of the form  $wy \Rightarrow$  it is active.

# The lower bound

Let  $k = \log_2 n$ . We will construct a sequence of hypercubes  $Q_0, Q_1, \dots, Q_{n/k}$  such that after moving according to the  $i$ -th orientation the ratio of  $k_i$ -passive vertices in  $Q_i$  is at most  $i/2^k$ .

$Q_i$  will have dimension  $n - ki$  and will consist of vertices starting with some string of length  $ki$ .

$Q_i$   $Q_{i+1}$



$A_w$   $ki$ -active vertices after phase  $i$

$N_w$   $k(i+1)$ -passive vertices after phase  $i+1$

$$2^l \geq |N_w| > 2^{l-1}$$

$Q_i$   $Q_{i+1}$



$k_i$

$k$

$$2^l \geq |N_w| \geq 2^{l-1}$$



Cannot both  
be in  $A_w$

Both  
are in


$N_w$

$$|A_w| \leq 2^{k-1}$$

$Q_i \quad Q_{i+1}$

$$|N_w| - (2^k - |A_w|) \leq 2^1 + 2^{k-1} - 2^k \leq 1$$

#(  $k(i+1)$ -passive vertices in  $Q_i$  after  $(i+1)$ -st phase ) - #(  $ki$ -passive vertices in  $Q_i$  after  $i$ -th phase )  $\leq 2^{n-k(i+1)}$



$M$

$$M/2^{n-ki}$$

$$((M + 2^{n-k(i+1)})/2^k)/2^{n-k(i+1)}$$

$$+(1/2^k)$$



# Questions:

Can the gap be improved?

Better lower bound for  
plain-buffer technique?