

Homework problems:

7.1 (due Oct 26, 2006) We are given an array $X[1..n]$ of real numbers (note that the numbers do not need to be positive). We want to find a segment of the array with the maximum sum. Formally, we want to find $a, b \in \{1, \dots, n\}$, $a \leq b$ such that

$$\sum_{i=a}^b X[i]$$

is maximum. Give an $O(n)$ algorithm which finds such a and b .

7.2 (due Oct 26, 2006) We are given n rectangles of sizes $a_1 \times b_1, \dots, a_n \times b_n$. We want to build the highest tower out of the rectangles. In a tower, if a rectangle of width w is on top of a rectangle of width w' then we require $w \leq w'$. We are allowed to rotate the rectangles (i. e., an $a \times b$ rectangle can be changed into a $b \times a$ rectangle). Give an $O(n)$ algorithm which finds the height of the highest tower.

(For example if the input is $11 \times 11, 8 \times 2, 1 \times 10$ then the solution is a tower of height $29 = 11 + 8 + 10$.)

Bonus problems:

7.3 (due Oct 26, 2006) Suppose that in Problem 7.2 we change the requirement $w \leq w'$ to $w < w'$, i. e., a rectangle on top of another rectangle has to be strictly thinner. Note that now it can happen that not all rectangles get used (e. g., if we have two squares of the same size). Give an efficient algorithm for the modified problem.

(For example if the input is $2 \times 11, 2 \times 10, 10 \times 10$ then the solution is a tower of height $22 = 2 + 10 + 10$.)

7.4 (due Oct 26, 2006) In this problem we want to find a large group of students which has a good average grade in both english and mathematics. There are n students numbered $1, \dots, n$. We are given two arrays $E[1..n]$, $M[1..n]$ of positive real numbers and a threshold T . We want to select a subset of the students $S \subseteq \{1, \dots, n\}$ such that the average grade of the group in english and in mathematics is at least T . Formally we require

$$\sum_{i \in S} E[i] \geq T \cdot |S| \quad \text{and} \quad \sum_{i \in S} M[i] \geq T \cdot |S|. \quad (1)$$

We want to find the largest group S which satisfies (1). Give an efficient algorithm for the problem.