The group on point for each problem are listed at its start. But if you have time, do look also at the other groups’ problems; they might be challenging and fun! It would not be wildly shocking if these problems were related to some problems that might be on the midterm-like homework, HW 5.

1 Cute Problem (... Well, Problem with a Cute, Smile-Inducing, Satisfying Solution)

1. [Thurs-late] You might have done this at your 2/1 or 2/2 workshop:

   \[L^r = \{\text{all strings } x \text{ such that } x^r \in L\}\]. For example, if \(L = \{1100, 1110\}\) then \(L^r = \{0011, 0111\}\). Prove that if \(L\) is regular, then so is \(L^r\). Do so by building an NFA that basically goes backward from a (guessed) final state to try to end up at the original start state.

   But given what you now know, can you see a much more beautiful proof that the regular sets are closed under reversal—one that doesn’t require you to get down and hack 5-tuples and machines? (Hint: Think “structural induction” and regular expressions!) (Since each group has to do two problems, you don’t have to give an utterly detailed proof, but please do try to see the key idea/approach.)

2 Moderately Easy Problems but on Relatively New-to-Us Tool

1. [Thurs-early] Let our alphabet be \(\{a, b, c, d\}\). Let \(L = \{ab^n c^n d \mid n \geq 0\}\). Prove (using the pumping lemma) that \(L\) is not regular. (Be careful. Make sure to not forget the case(s) where \(y\) is \(a\) or contains \(a\) and some \(b\)’s!)

2. [Wed-late] Consider trying to use the pumping lemma to show that the following language over the alphabet \(\{0, 1\}\) is not regular: \(L = \{w \mid w\text{ has the same number of }0\text{'s and }1\text{'s in it}\}\). Professor Foo tries to do this by a standard pumping lemma proof. So Foo lets \(p \geq 1\) be an arbitrary natural number. And then if \(p\) is even lets \(w \in L\) be \((01)^{p/2}\) and if \(p\) is odd lets \(w \in L\) be \((01)^{\lfloor p/2 \rfloor}0\). Will this lead to a successful proof? Or is the professor doomed, and if so, why? (Yes, if you were listening carefully you would have heard me touch in precisely this issue in class on Wednesday; but either way, this problem is still worth doing, as it makes the point that one’s choices, such as of \(w\), require great care.)
3 Harder Problem (But with Luck Easy for You by Now)

1. [Wed-early-Shir] Draw a DFA accepting the following language, over the alphabet \{0, 1, 2, 3, 4, 5, 6\}: All strings that when interpreted as base-7 integers are multiples of 6. (Ignore leading 0’s. Treat the empty string as if it is interpreted as the value 0. So 00000 represents 0 and 00011 represents the decimal number 8 and 101 represents the decimal number 50.) (Hint: A good way to do this type of thing is to be inspired by long division. The reason I say that that is good is the long-division inspired attack gives you a way to more generally solve division problems, across differs bases and across whatever positive integer you want to divide by and so on. I myself think of this in the slightly more hands-on way I’ve approach it in class as that is quite hands-on and clear; but do note that what I am doing is exactly what one does in long division.)

4 Harder Problem (Outright Challenging if You Decide to Not Read the Hint and Do It Purely Yourself, Which Could Be Fun!)

1. [Wed-early-Mikayla] You now know enough to prove that first thirds of regular sets are regular, and that 3rd 7ths of regular sets are regular, and lots more. Are there no limits to what we can do along the lines of such “fractional parts of regular languages” problems? Let us see! Let \( \Sigma = \{0, 1, 2\} \). Let \( A \) be an arbitrary regular set over alphabet \( \Sigma \). Consider the language of strings in \( A \) with the middle thirds cut out: \( L_A = \{w \mid (\exists x, y, z)[|x| = |y| = |z| \land w = xz \land xyz \in A]\} \). Clearly that there exists a regular set \( A \) such that \( L_A \) is regular, e.g., if \( A = 0^* \) then \( L \) is \((00)^*\). Prove that there is a regular set \( A \) such at \( L_A \) is *not* regular. (Hint: Consider the regular set \( W = 0^*122^*10^* \). Cleverly choose a regular set to intersect with \( L_W \), so as to have the intersection be something that you can easily, using the pumping lemma, prove nonregular; and then do prove that intersection nonregular, and then draw your desired conclusion about \( L_W \). Yeah!)