Note: There are 4 problems on this quiz. (On any quiz, some problems may be on the back of the page. So always look at the back of the page, even if there is no note such as this mentioning that or mentioning the number of problems. This is probably the last time we’ll explicitly mention on a quiz that there is a problem on the back of the page.) Note: The points on a problem don’t necessarily correspond to its difficulty, e.g., on this quiz, Question 4 is harder than Question 3.

**Question 1. [10 points]** In class, on a slide last class—the slide that had (as noted on the slide just before it) the rabbit’s head almost exploding because such a huge result has such a short, magically delightful proof!—we proved, by diagonalization, that

(fill in the blank: ) is not an RE set.

**Question 2. [25 points]** You may (if you got Question 1 right) draw in this problem on the result covered in Question 1 above. On the previous quiz, many people thought, and used, the flawed claim: “If $A$ is r.e. and $B \subseteq A$, then $B$ is r.e.” Shatter that claim. That is, let $\Sigma = \{0, 1\}$ and give sets $A \subseteq \Sigma^*$ and $B \subseteq \Sigma^*$, such that $A$ is r.e., $B \subseteq A$, yet $B$ is not r.e.

**Question 3. [35 points]** Recall that for $A \lesssim_m B$ (i.e., (recursive) many-one reduction) is defined as follows: We say $A \lesssim_m B$ iff ($\exists$ recursive function $\sigma$)(\forall x)[x \in A \iff \sigma(x) \in B].$ Recall that HP = \{ $x$ | $x \in L(M_x)$ \} or, equivalently, \{ $x$ | $M_x$ on input $x$ accepts \}. Let $L_4 = \{ xx \ | \ x \in \text{HP} \}$ (note: $xx$ denotes $x$ concatenated with $x$, e.g., if $x = 001$ then $xx = 001001$). Prove that $L_4$ is undecidable by explicitly giving a recursive many-one reduction from HP to $L_4$ (which includes of course arguing that the function you give is indeed a recursive many-one reduction from HP to $L_4$). (Of course, make sure to give a reduction from HP to $L_4$ and not the other way around.)
Question 4. [30 points] Let $L_5 = \{ \langle i, j \rangle \mid L(M_i) = L(M_j) \}$. Prove that $L_5$ is undecidable by explicitly giving a recursive many-one reduction from HP to $L_5$ (which includes of course arguing that the function you give is indeed a recursive many-one reduction from HP to $L_5$). (Of course, make sure to give a reduction from HP to $L_5$ and not the other way around.)