For each of the three problems, give a many-one (recursive) reduction function \( \sigma \) that does the reduction, and include the 3-part argument that it does the reduction.

**Question 3. [35 points]** Show \( HP \leq_m B \), where

\[
B = \{ i \mid L(M_i) \text{ is infinite} \}.
\]

**Question 2. [30 points]** Professor Bar claims that for every pair of recursive sets, \( A \) and \( B \), it holds that \( A \leq_m B \). Prove or disprove Professor Bar’s claim, and you must start your answer by explicitly stating “I will prove” or explicitly stating “I will disprove.” (Hint: Both \( \emptyset \) and \( \{ x \in \{0, 1\}^* \mid |x| \text{ is even} \} \) are recursive sets.)
Question 3. [35 points] Show $\mathcal{HP} \leq_m B$, where

$$B = \{ i \mid L(M_i) \text{ is infinite} \}.$$