CSC 286/486 Group A Homework

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Submission

Please submit a hard copy in class on Monday November 7, or email a PDF or scanned homework, by 2PM that day, to all of lbeadle@cs.rochester.edu, kdharsee@ur.rochester.edu, dfink2@u.rochester.edu, gkim21@cs.rochester.edu, jkottas@u.rochester.edu, xlian2@cs.rochester.edu, srazavi@cs.rochester.edu, and csc486staff@cs.rochester.edu.

Grading and Points

Each problem is worth 20 points, for a total of 100 points. Partial credit will be given for partial solutions.

Problem 1

Prove TALLY is a strict subset of SPARSE, where TALLY = \{X | X is a tally set\} and SPARSE = \{X | X is a sparse set\} (remember to show both the ‘subset’ part and the ‘strict’ part).

Problem 2

We saw in class that two definitions for a set S being sparse are equivalent: one that bounds the number of number of strings in S of length ≤ n, and another that bounds the number of strings in S of length exactly n.

Prove or disprove: The following two definitions for a set D being dense are equivalent (note the first one is our true definition):

1. There is a linear-exponential function f such that \(\exists N \forall n > N \|\{x \in D \mid |x| \leq n\}\| \geq f(n)\).
2. There is a linear-exponential function f such that \(\exists N \forall n > N \|\{x \in D \mid |x| = n\}\| \geq f(n)\).

For this problem, a linear-exponential function is one that can be written in the form \(f(n) = CB^n\), where \(C, B \in \mathbb{R}\) and \(C > 0\) and \(B > 1\).

Problem 3

(A) What is the census function of \(\{0,1\}^*\), where \(c(n)\) is the number of strings in \(\{0,1\}^*\) with length ≤ n?

(B) Is the following set \(L\), for some fixed integer \(k \geq 10\), sparse, dense, neither, or both?

\[L = \{\text{binary representation of } n \mid n \text{ is divisible by } 2^k\}\]

Problem 4

Construct a set that is neither sparse nor dense (careful with the definition of dense).

Problem 5

Show that \(L\) is self-reducible, where

\[L = \{\#x_1\#x_2\#\ldots\#x_k\# \mid [k > 0] \land [\forall 1 \leq i \leq k, x_i \in M] \land \left[\exists C \subseteq \{1,2,\ldots,k\}, C \neq \emptyset \land \sum_{c \in C} x_c = 0\right]\},\]

where \(M\) is the following encoding of the integers: the encoding of \(z\) is \(SB\), where \(S = +\) if \(z \geq 0\) and \(-\) otherwise, and \(B\) is the standard binary encoding of \(|z|\) (no leading 0’s, so the full encoding of 0 is +).