Group B Quiz 1 Answers
Sayak Chakraborti, Andrii Osipa, Parker Riley, Zhuojia Shen, Evan Strohm, and Hassler Thurston
November 16, 2016

Question 1: (40 points)
State whether the following statements are True or False:

(a) If \( P \neq NP \), then for any one-way function \( f \), \( g(x) = 1 \cdot f(x) \) is also a one-way function. \( \text{True} \)

(b) The function \( f(x) = 1^{\log(\log(|x|))} \) is honest. \( \text{False} \)

(c) The function \( f(x) = 1 \cdot x \) is honest. \( \text{True} \)

Question 2: (20 points)
Suppose \( P \neq NP \) and one-way functions exist. Suppose \( f \) is a one-way function. Let \( f' \) be a function where

\[
f'(x) = \begin{cases} 
  f(a) \cdot f(a) & \text{exists such that } x = a \cdot a, \\
  0 & \text{otherwise}.
\end{cases}
\]

Is \( f' \) also a one-way function? If so, begin your answer with “Yes” and then address specifically all three conditions for being one-way, namely: honesty, polynomial-time computability, and polynomial-time noninvertibility. If not, begin your answer with “No” and then state which condition is violated and explain how it is violated.

**Answer:** \( f' \) is clearly honest and polynomial-time computable. Suppose that it were polynomial-time invertible, and that \( h' \) were the function inverting it. We could then construct an inverse for \( f \) as follows:

\[
f^{-1}(x) = \begin{cases} 
  a & h'(x \cdot x) \text{ is of the form } a \cdot a \text{ for some string } a, \\
  \text{undefined} & \text{otherwise}.
\end{cases}
\]
Question 3: (40 points)

Fill in the blanks below to complete the statements proven in the last class:

1. One-to-one one-way functions exist if and only if $P \neq \text{UP}$.

2. For each $k \geq 2$, $k$-to-one one-way functions exist if and only if $P \neq \text{UP}_{\leq k}$. 