Question 1. [10 FAKE (not real; does not count) points]
Write a simplified expression (not a summation) that gives the number if binary strings of length less than or equal to \( n \):

Answer:

Question 2. [90 FAKE (not real; does not count) points] Let us define \( HP = \{ i \mid i \in L(M_i) \} \). You may assume that \( M_1, M_2, M_3, \ldots \) is a standard enumeration of Turing machines and that \( HP \) (using exactly the definition above) is undecidable (we implicitly use the standard 1-to-1 bijection between \( N \) and \( \Sigma^* \) to treat \( i \) both as integer and string).

Prove that \( A = \{ i \mid L(M_i) \neq \emptyset \} \) is undecidable. (Do not invoke Rice’s Theorem. Rather, either use the so-called contradiction method or, better probably, prove (in doing that, one part will be to GIVE the reduction, and another will be to cover the three elements of proving that the function you gave in fact IS a reduction between these two sets) that \( HP \) reduces to \( A \) by some recursive many-one reduction (aka recursive mapping reduction).)

(Side comment: This is an usually easy problem, since we don’t have much time for this quiz. But you should also be able to solve such harder problems as proving that, for example, \( A = \{ i \mid L(M_i) \text{ is an infinite language} \} \) is undecidable. But on this quiz, I’m asking NOT about that but about Question 2.)

Answer: