Question 1. [40 points]

We will use $\cdot$ to denote concatenation, e.g., $000 \cdot 111 = 000111$. Prove that if $A \subseteq \{0, 1\}^*$ is undecidable then $L_A$ is undecidable, where $L_A = \{101010 \cdot x \mid x \in A\}$. (Hint: You may take it as known that if a set $B$ is undecidable, then any set $C$ such that $B$ (recursive) many-one reduces to $C$ (which equivalently means that $B$ (recursive) mapping reduces to $C$) is undecidable.)
Question 2. [45 points]

Let $HP = \{ i \mid i \in L(M_i) \}$. Assume that $M_1, M_2, M_3, \ldots$ is a standard enumeration of Turing machines (we use the standard 1-to-1 bijection between $\mathbb{N}$ and $\Sigma^*$ to treat $i$ as both an integer and a string) and that we have already proven that set, $HP$, to be undecidable.

Prove that $A = \{ i \mid L(M_i) = \{ a^n b^n \mid n \in \mathbb{N}^+ \} \}$ is undecidable. (Note: In this course, $\mathbb{N} = \{ 0, 1, 2, \ldots \}$ and $\mathbb{N}^+ = \{ 1, 2, \ldots \}$.) You may use either the so-called “contradiction method” (a.k.a. “Turing reduction” method, as taught in CSC 2/480) or, better, give a many-one (mapping) reduction from $HP$ to $A$, i.e., show that $HP \leq_m A$. (Although we would prefer a many-one reduction since that is the method we will be using throughout this class, either method, if done correctly, will get you full credit on this quiz. Use the method you’re most confident with to ensure you get a correct answer down in the allotted time.)

Question 3. [15 points] The “Invitation” of the preface of the Hemaspaandra-Ogihara textbook makes the surprising claim that every “student of computer science already possess the ability required to understand, enjoy, and employ complexity theory.” It makes this claim because the book’s view is that at the heart of complexity theory is:

(fill this in, either in the version of secret 1 of the invitation or in the version of secret 2).