Quiz Answer Sketches  
CSC 286/486: Computational Complexity  
Fall 2017  
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This adds up to 110 points, so there are 10 extra credit points possible here. (That is, if you get 105 points, your score on the quiz will be 105/100.)

For the first two problems, let us take it that HP is encoded over the alphabet \{0,1\} and that all the sets these problems are speaking of must be over the alphabet \{0,1\}. Also, we say that A and B have a recursive separator if there is a recursive set C such that $A \subseteq C$ and $C \subseteq \overline{B}$ (that is, all elements of A are members of C, yet no elements of B are members of C).

**General note:** often on answers or answer sketches, I will not worry about always putting math variables into italics, e.g., I might write: If A satisfies $A = \emptyset$. You can see this at work in some places in the answers on this set, e.g., the answer sketch for Question 1.

**Question 1. [40 points]**

Prove that every two disjoint (i.e., their intersection is empty) coRE sets have a recursive separator.

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Answer Sketch: Let A and B be disjoint coRE sets. Let EA and EB be enumerators for their complements respectively; since A and B have RE complements, we know that we have such enumerating TMs. Here is our algorithm (that always halts) for a recursive separator C. On arbitrary input y, our algorithm runs the enumerators in a dovetailed fashion (e.g., it runs EA for one step, then EB for one step, then it runs EA for two step, then EB for two steps, and so on), and as soon as one of the enumerators enumerates y, we accept if EB enumerated y and we halt and reject if EA enumerated y. (Informally, we have them race to output y.) Since A and B are disjoint, the union of their complements is \( \Sigma^* \), and so one or both of EA and EB will accept on each input. So we will always halt. And for each string in A it will be EB that enumerates it and for each string in B it will be EA that enumerates it, so C does satisfy $A \subseteq C$ and $C \subseteq \overline{B}$. 

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**Question 2.** [40 points] Prove that there exist two disjoint RE sets, $A$ and $B$, that have no recursive separator. You may draw on the fact that $HP = \{ x | M_x(x) \text{ accepts} \}$ is undecidable.

Answer Sketch: Let $B = \{ x | M_x(x) \text{ accepts} \}$. Let $A = \{ x | M_x(x) \text{ halts and rejects} \}$. $A$ and $B$ are both RE sets and are disjoint. If $C$ is a recursive separator for them, with $A \subseteq C$ and $C \subseteq \overline{B}$, then for some $k$ for which $M_k$ is a total TM, $L(M_k) = C$. Since $M_k$ is total, $M_k(k)$ must be in $A \cup B$. But if it is in $A$ then by the def. of $A$ that means $M_k(k)$ halts and rejects, but $A \subseteq C$ so $k$ must be in $C$, so since $M_k$ is deciding $C$ that means $M_k(k)$ accepts; but $M_k(k)$ cannot both accept and halt and reject, so we have reached a contradiction when $k$ is in $A$. And if $k$ is in $B$ then by the def. of $B$ that means $M_k(k)$ accepts, yet since $C \subseteq \overline{B}$ and $M_k$ is a decider for $C$ we must have that $k$ is not in $C$ and so $M_k(k)$ cannot accept; so we have also reached a contradiction when $k$ is in $B$. Since we know that $k$ is in $A$ or is in $B$, we have contradicted our assumption that there exists a recursive separator for them.

**Question 3.** [5 points] Answer True or False: It is cheating to post to online forums (such as StackOverflow, etc.) seeking answers to in-progress take-home quizzes:

Answer: True.

**Question 4.** [10 points]

The “Invitation” of the preface of the Hemaspaandra-Ogihara textbook makes the surprising claim that every “student of computer science already possess the ability required to understand, enjoy, and employ complexity theory.” It makes this claim because the book’s view is that at the heart of complexity theory is:

(fill this in, either in the version of secret 1 of the invitation or in the version of secret 2).

Answer: “algorithms” or “simple algorithms.”

**Question 5.** [5 points] State (no points for this but if you don’t get it correct you will get a 0 on this problem) which one of the 286 and 486 numbers you are registered for this course under:

and then state the number of 100-point chunks that will be dropped for each person taking the course under that number:

Answer: [whichever applies to you]; if 286 the right answer is 8 and if 486 the right answer is 3.