Take-Home, Part-Graded/Part-Nongraded Exercise (Brief Answers to the Graded Ones) 2019/09/18
CSC 286/486: Computational Complexity Fall 2019
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Rules: The rules for take-home exercises hold. Talk only with your groupmates. But you may use books/internet etc., except not to seek to look up the answer. Our slides are totally fair to use. Hand in this sheet, one per group at 2PM, 9/23, as Q1 will be graded. Q2, though, will not be graded—not even what you put up at the white board. Q2 can be hard, so do not worry if you cannot solve it—but do put in an hour or two trying, as you might, and even if you don’t, having tried will help you better understand solutions when you see them.

Question 1. [30 points]
Show $A \leq_m B$, where the alphabet $\Sigma = \{0, 1\}$, and

$A = \{x \in \{0, 1\}^* \mid |x| \text{ is even}\}$

$B = \{x \in \{0, 1\}^* \mid |x| \text{ is odd}\}$.

Answer:

Let $\sigma(x) = 0x$, or in other words, its input with a zero concatenated onto the front.

$\sigma$ is clearly a recursive function.

If $x \in A$, $|x|$ is even. Thus $|0x|$ is odd, and so $0x \in B$. Thus $\sigma(x) \in B$.

If $x \notin A$, $|x|$ is odd. Thus $|0x|$ is even, and so $0x \notin B$. Thus $\sigma(x) \notin B$.

We have that $x \in A \iff \sigma(x) \in B$, and we mentioned that $\sigma$ is a recursive function; and so we have that $A \leq_m B$.

Question 2. [70 points] Show $\overline{HP} \leq_m B$, where

$B = \{i \mid \overline{L(M_i)} \text{ is infinite}\}$.

Answer:

$\sigma(i) = j$ where $j$ is the index of a Turing machine $M_j$ that does the following on input $y$: Simulate $M_i$ on $i$.

$\sigma$ is clearly a recursive function.

If $i \in \overline{HP}$, $M_i$ rejects $i$. Thus $M_j$ accepts no strings, and so $L(M_j) = \emptyset$ and $\overline{L(M_j)} = \Sigma^*$, which is infinite. Thus $j \in B$.

If $i \notin \overline{HP}$, $M_i$ accepts $i$. Thus $M_j$ accepts all strings, and so $L(M_j) = \Sigma^*$ and $\overline{L(M_j)} = \emptyset$, which is finite. Thus $j \notin B$.

We have that $i \in \overline{HP} \iff \sigma(i) \in B$, and we have that $\sigma$ is a recursive function. So $\overline{HP} \leq_m B$.

Question 3. [nongraded]
Prove that there exists a pair of disjoint RE sets, $A$ and $B$, such that for no recursive set $C$ does it hold that $A \subseteq C \subseteq B$.

(Note: By doing this, you will have proven what in the jargon is typically expressed by: There exists a pair of disjoint r.e. sets that are recursively inseparable. In contrast, in class you saw that every pair of disjoint co-r.e. sets are recursively separable!)

Answer:

[We’ll go over this in class.]