Note: This is in effect an errata sheet regarding the proof of Mahaney’s Theorem that appears in Bovet-Crescenzi, in particular regarding page 88 there in the proof of Theorem 5.6. In particular, this replaces the first 27 lines of that page (and the changes are marked here in red).

Notice that if \( k = c_S(m) \), then for each unsatisfiable \( y \) of length, at most, \( |x| \), \( g(h(y), k, 0^m) \) must be included in \( S \) and we have obtained a reduction from SATISFIABILITY to \( S \) for words of up to length \( |x| \). If, on the other hand, \( k \neq c_S(m) \), then we cannot be certain that \( g(h(y), k, 0^m) \) belongs to \( S \).

For each \( n \) and for each \( k \leq p(p_h(n)) \), let us define \( f_{n,k}(y) = g(h(y), k, 0^{p_h(n)}) \). Clearly, for each \( k \), function \( f_{n,k} \) is computable in polynomial time with respect to the length of \( y \), that \( |y| \leq n \). Furthermore, it follows from the previous considerations that \( y \) is not satisfiable if and only if \( f_{n,c_S(p_h(n))}(y) \in S \).

If \( k = c_S(p_h(n)) \), then a constant \( c_1 \) depending on \( p \) and an integer \( n_0 \) exists such that \( |\langle h(y), k, 0^{p_h(n)} \rangle| \leq 2p_h(n) + c_1 \log(p_h(n)) \leq 3p_h(n) \), for all \( n \geq n_0 \). The unsatisfiable formulas of length, at most, \( n \) are then transformed from \( f_{n,k} \) in, at most, \( p(p_g(3p_h(n))) \) distinct words of \( S \), for each \( n \geq n_0 \).

The tree-visiting algorithm described in the proof of Theorem 5.6 is thus modified so that all possible pruning functions \( f_{n,k} \) with \( k \leq p(p_h(n)) \) are considered. If the revised algorithm visits more than \( |x|p(p_g(3p_h(|x|))) + |x| - 1 \) inner nodes, then it infers that \( k \neq c_S(p_h(n)) \) and starts to consider the next value of \( k \).

The new algorithm is thus the following:

\[
\begin{align*}
\text{begin} & \{\text{input: } x\} \\
& \text{for } k = 0 \text{ to } p(p_h(|x|)) \text{ do} \\
& \text{begin} \\
& \quad \text{execute the tree-visiting algorithm described in the} \\
& \quad \text{proof of Theorem 5.6 using } f_{|x|,k} \text{ as a pruning function} \\
& \quad \text{and visiting, at most, } |x|p(p_g(3p_h(|x|))) + |x| - 1 \text{ inner nodes;} \\
& \quad \text{if the algorithm accepts then accept;} \\
& \quad \text{reject;} \\
& \text{end;} \\
\text{end.}
\end{align*}
\]