



Control in the Presence of Manipulators: Cooperative and Competitive Cases

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Motivation

Manipulation and control attacks on elections have, separately, been extensively studied. But in the real world, both types of attacks may occur together, collaboratively or competitively.

Definitions

We examine several natural election systems including:

- **Approval:** Each vote is a 0-1 vector indicating disapproval or approval for each candidate. The candidate(s) with the most approvals win.
- **Condorcet:** p is a winner if p strictly beats every other candidate in a pairwise election between them.
- **Plurality:** Each voter gives one point to his or her top choice and zero to the remaining candidates.
- **Veto:** Each voter gives zero points to his or her bottom choice and one point to the remaining candidates.
- **Borda:** Each voter gives $m - 1$ points, where m is the size of the candidate set, to his or her top choice, $m - 2$ to his or her second choice, and so on through giving no points to his or her bottom choice.

Controlling Elections

The “Chair” Doing the Control Action May

- **Collaborate with the Manipulators.**
 - The election chair and the manipulators have the same preferred (or despised) candidate.
- **Compete with the Manipulators.**

We Study Two Cases:

- **CF:** The election chair acts first.
- **MF:** The manipulators act first.

Who goes first, the chair or the manipulators, can have an effect on the corresponding complexity of control.

Upper-Bound Results

These results hold for all election systems, \mathcal{E} , having a polynomial-time winner problem.

Problem	CF	CF-revoting	MF	MF-revoting
$\mathcal{E} - \begin{bmatrix} C \\ D \end{bmatrix} C \begin{bmatrix} A \\ D \end{bmatrix} \begin{bmatrix} C \\ V \end{bmatrix}$	NP ^{NP}	N/A	coNP ^{NP}	N/A
$\mathcal{E} - \begin{bmatrix} C \\ D \end{bmatrix} C \begin{bmatrix} PC \\ RPC \\ PV \end{bmatrix} - \begin{bmatrix} TE \\ TP \end{bmatrix}$	NP ^{NP}	NP ^{NP}	coNP ^{NP}	coNP ^{NP}

Conclusion: Although these are natural problems, their complexities fall at higher levels of the polynomial hierarchy.

Unweighted-Election Results

The three systems were examined for comparison with previous work on control and because their manipulation problems are in P.

Problem	Approval			Condorcet			Plurality		
	Collab.	CF	MF	Collab.	CF	MF	Collab.	CF	MF
$\begin{bmatrix} C \\ D \end{bmatrix} CAC$	P	P	P	P	P	P	NPC	NPC	NPC
$\begin{bmatrix} C \\ D \end{bmatrix} CDC$	P	P	P	P	P	P	NPC	NPC	NPC
$\begin{bmatrix} C \\ D \end{bmatrix} CAV$	NPC P	NPC P	NPC P	NPC P	NPC P	NPC P	P	P	P
$\begin{bmatrix} C \\ D \end{bmatrix} CDV$	NPC P	NPC P	NPC P	NPC P	NPC P	NPC P	P	P	P
$\begin{bmatrix} C \\ D \end{bmatrix} CPC-TE$	P	P	P	P	P	P	NPC	NPC	NPC
$\begin{bmatrix} C \\ D \end{bmatrix} CPC-TP$	P	P	P	P	P	P	NPC	NPC	NPC
$\begin{bmatrix} C \\ D \end{bmatrix} CRPC-TE$	P	P	P	P	P	P	NPC	NPC	NPC
$\begin{bmatrix} C \\ D \end{bmatrix} CRPC-TP$	P	P	P	P	P	P	NPC	NPC	NPC
$\begin{bmatrix} C \\ D \end{bmatrix} CPV-TE$	NPC P	NPC P	NPC P	P	P	P	P	P	P
$\begin{bmatrix} C \\ D \end{bmatrix} CPV-TP$	NPC P	NPC P	NPC P	P	P	P	NPC	NPC	NPC

Conclusion: The addition of manipulators does not change the P complexity cases in the above important systems: The NPC cases are already NPC without that addition.

Weighted-Election Results

Problem	3-Candidate Veto			3-Candidate Borda		
	Collab.	CF	MF	Collab.	CF	MF
CCAV	NPC	P	P	P	NPC	NP-hard and coNP-hard
CCDV	NPC	P	P	P	NPC	NP-hard

Conclusion: The addition of manipulators to elections with weighted votes sometimes increases the complexity of control.

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