

A Study of Linear Variable Differential Transformers in MATLAB

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Abstract

*The purpose of this lab was to test Faraday's Law using a Linear Variable Differential Transformer (LVDT). Basically, an LVDT consists of three coils (of 100 windings each) spaced evenly along an insulating rod. A voltage of around 6 V was applied across the inner coil, and a ferromagnetic rod was inserted into the solenoids. According to Faraday's Law, a changing magnetic flux within the ferromagnetic core will induce an output voltage in the outer coils. Due to the winding reversal between the coils though, the induced voltage will be opposite. Thus, a zero point (where the induced voltages cancel out) can be found using MATLAB's **softscope** oscilloscope function. Furthermore, as the core is moved linearly between the windings, a linear shift from a negative voltage to a positive voltage can be found. This linear fit is caused by changes in the exterior windings enclosing the core, and a linear function can be found to describe this phenomenon using MATLAB's **polyfit** command, which generated a slope of 6.5 mV per cm. A voltage decrease at the LVDT's ends can be attributed to the core partially leaving the inner coil. In conclusion our data agreed with the theory, but small changes-such as a longer core-could have generated less error and produced a stronger linear fit.*

1 Introduction

In this lab, a Linear Variable Differential Transformer (LVDT) was made and used with the National Instruments 6009 USB analog to digital converter to find the displacement of a semiconductor core inside a PVC tube. This project also involved using MATLABs softscope tool to gather and display data.

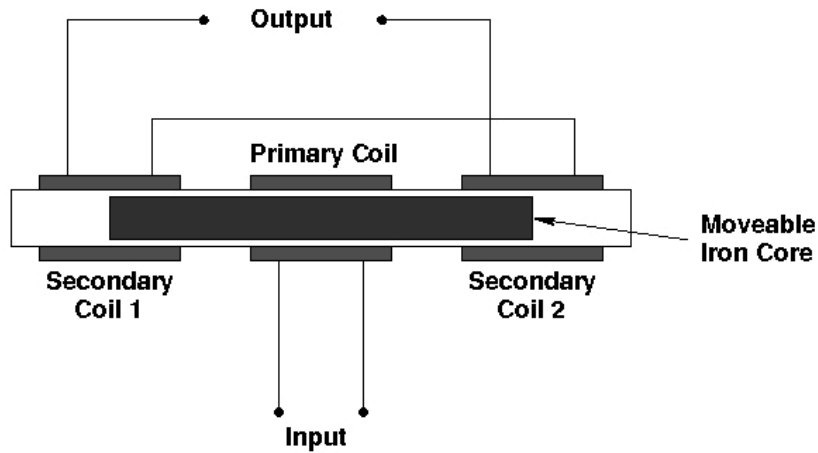
The LVDT consisted of three coils of copper wire wound around a PVC tube with the center primary coil powered by an AC source. The outer secondary coils were connected in reverse series to each other and an output voltage from each coil was sent to the 6009 to be converted to a digital signal and recorded. A schematic of the setup is shown below in Figure 1.

To measure the displacement, the semiconductor core was moved from the left end of the tube to the right end. As the coil was moved from left to right, the number of turns changed and an induced emf was produced that opposed the motion of the core. This emf was linearly related to the core by the equation shown below.

$$\epsilon = -\frac{N}{L} \frac{d\Phi}{dt} x,$$

*Department of Electrical and Computer Engineering, *University of Rochester*, Rochester, NY 14627

†Department of Physics and Astronomy, *University of Rochester*, Rochester, NY 14627



Schematic drawing of an LVDT

Figure 1: Diagram of a typical linear variable differential transformer.

where L is the length of the solenoid (inductor), ϵ is the induced voltage, and Φ is the magnetic flux. X is a generalized coordinate specifying the position of the semiconductor core. From this equation, the displacement can be calculated using the data collected for voltage.

2 MATLAB Scripts

A MATLAB script was first used to find the position of the core where no voltage was produced. This point was then marked. The script first initialized the analog input channels using the `analoginput()` and `addchannel()` commands. Then the `softscope` command was used to pull up a window displaying the voltage output as a function of time.

To collect data on the voltage output as the core was moved, the `getdata()` command was used in a for-loop structure. The sample rate was set to 600 samples per second using the `setverify()` command, and the number of samples per trigger was set to 6000 using `set()`. The root mean square voltage values for each coil were also calculated for each core position inside the for-loop and stored in a matrix. After this data had been calculated, the `display` and `pause` commands were used to let the user know when to move the position of the core.

After the data was collected, a separate script was used to plot the RMS voltage as a function of the displacement of the semiconductor core using MATLAB's `plot` function.

3 Discussion

A Linear Variable Differential Transformer (LVDT) consists of three 100 loop coils. Our LVDT, however, had 100 windings for the outer two coils, and 90 windings for the inner winding. This was done for two reasons. One, the geometry of our device yielded a symmetric result with 90 inner windings; and two, the number of windings in the central coil does not *technically* matter. All that matters is that a constant flux is generated throughout the iron core. Since the number of windings for the outer coils is equivalent, the induced flux will be equal and opposite at the zero point.¹

¹The zero point is the point at which the iron core is centered within the LVDT.

Zero point: Softscope Output

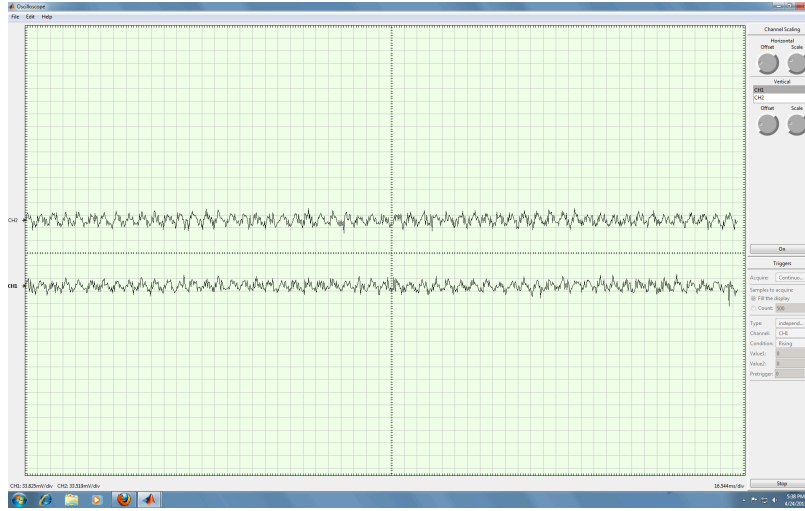


Figure 2: Investigating the zero point of our detector. The two outputs correspond to the two input channels.

As you can see the data generates more of a sinusoid than a linear trend. This curvature will be explained shortly. The inner linear portion is shown in the second figure.

A 6V sinusoidal input was generated across the inner coil, and the two outputs-at either end of the LVDT-were connected to the first and second analog input on the DAQ 6009. The DAQ inputs transmitted data, specifically 6000 data points sampled at 600 points per second, to the computer for each test run. The iron core was initially positioned at one end of the LVDT, and then systematically moved through the transformer. Each test, and thus data set, corresponded to a different placement of the iron core.

Each test also generated a different V_{rms} value for each input. The RMS value was calculated using the following formula:

$$V_{rms} = \sqrt{\frac{1}{N} \sum v_i^2},$$

where N is the number of data points in each column. In order to find the appropriate difference between the outer coils, the rms values for each test were subtracted from each other. This resulting difference was plotted, and the graph is generated in Figure 3.

I will break the discussion of this data up into two sections. The first section will analyze the inner linear portion, and the second section will explain why the RMS voltage drops off at the end-creating a sinusoidal trend.

The inner portion, shown in Figure 4, was fitted linearly using MATLAB's **polyfit** function. The slope was found to be 6.5 mV/cm. Actually, the inner portion is composed of three lines. Starting from the left (-1 cm to 0 cm) the slope is different than the slope between 0 and 1. Furthermore, the slope between 1 and 2 is also unique. The slope on the left occurs when the iron core overlaps with the inner coil and the 'negative' coil on the left. The induced emf can be represented as such:

$$\epsilon = -N_{left} \frac{d\Phi}{dt},$$

where N_{left} represent the number of coils encompassing the iron core on the left (negative) end and Φ is the magnetic field (B) dotted with the cross sectional area (A). For this segment,

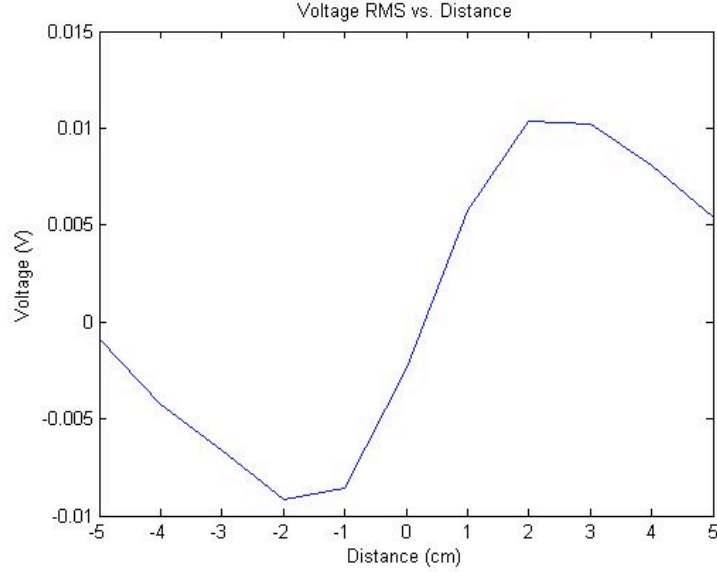


Figure 3: Voltage vs. Core Position: An interesting trend is produced above.

iron is between the inner coil at all times. The ferromagnetic material decreases linearly as a function of distance for the outer (left) coil. This is a linear expression dependent on the change in windings only.

The central line (in between 0 and 1) is composed of voltages induced by *both* outer coils. Consequently, a mathematical relationship for this is expressed below:

$$\epsilon = -(N_{left} - N_{right}) \frac{d\Phi}{dt}.$$

As you can see this slope, due to the addition of the right coil, will differ from the previous slope. Our data reflects this trend.

The reason this slope is steeper than the other two is because; one, the negative coils on the left side are decreasing while, two, the positive coils are increasing. Thus, the induced voltage is approaching a positive value twice as fast.

Finally, the right line has a more gradual slope for the same reason as the left slope.

$$\epsilon = -N_{right} \frac{d\Phi}{dt}.$$

Theoretically these two slopes should be equal, but inconsistencies in the windings introduced error into our system. In general, our data agrees with the theory.

The explanation for why the voltage drops off as the core (rod) approaches either end is actually simple. As the rod approaches the ends, the iron core leaves the inner coil. This results in a decreased flux amount since N_{inner} is now smaller. If the iron core leaves the inner coil completely then the induced voltage will go to zero since the magnetic field (generated by the inner coil inside of the iron) will disappear. In mathematical terms

$$\frac{d\Phi}{dt} = 0.$$

The point at which the voltage drops off by ten percent due to this effect occurs when ten percent of the inner coils (9 windings) do *not* overlap the iron core. This occurs between 2 and 3 cm on the right (positive) side, and between -1 and -2 cm on the the left (negative) side. The antisymmetric dowel rod generates this uneven behavior.

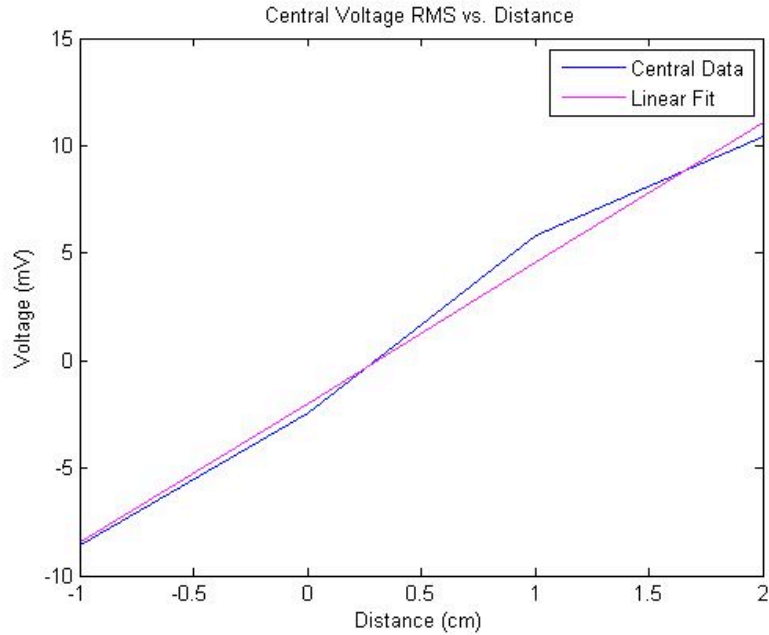


Figure 4: Voltage vs. Core Position: Only the central points which have a linear behavior are analyzed.

4 Conclusion

Our linear variable differential transformer generated linear behavior when the core was fully inserted into the inner coil. Once the core left the inner coil, which was responsible for driving the flux within the core, the induced flux decreased significantly-causing a plunge towards a zero induction value.

For the central, well behaved portion, the linear plot actually consisted of three separate lines. The slope of these lines varied according to the number of loops enclosing the iron core. The left side was enclosed by only the inner coil and the negative coil. At the zero point all three coils enclosed the core. Finally, when the core was moved to the right side the core was enclosed by only the inner coil and the positive coil.

It is also interesting to note how our LVDT varied from others; particularly in the number of inner windings. Instead of 100 windings, our inner coil contained 90 windings. This was both geometrically favorable (for our design) and theoretically consistent-though slightly less flux was induced.

This experiment should be repeated with a longer iron core. This would prevent variations in flux density, ensuring that a constant Φ would be present. This inconsistency forced us to test two variables *simultaneously*. This is not good science, and therefore should be fixed. Also, more care should have been taken while constructing our LVDT. Some of the windings overlapped, producing error in our results and unwanted noise in the Softscope output.

Overall, our data agreed with the theory, behaving linearly within the central limits of -1 and 2. Therefore, our test of Faraday's Law was successful.