CSC 252:
Data Representation

Bits, Bytes, and Integers

This Week's Action Items

- Read Chapter 2
- Finish Quiz 1 and 2 on Blackboard
- Start on Assignment 1
  - Finish Pre-Assignment 1 on Blackboard
    - Due Date: Thursday September 3 at noon

Recap: Bits, Bytes, and Integers

- Representing information as bits
- Bit-level manipulations
- Integers
  - Representation: unsigned and signed
  - Conversion, casting
  - Expanding, truncating
  - Addition, negation, multiplication, shifting
  - Summary
- Representations in memory, pointers, strings

Contrast: Logic Operations in C

- Contrast to Logical Operators
  - &&, ||, !
    - View 0 as "False"
    - Anything nonzero as "True"
    - Always return 0 or 1
    - Early termination
- Examples (char data type)
  - !0x41 -> 0x00
  - !0x00 -> 0x01
  - !!0x41 -> 0x01
  - 0x69 && 0x55 -> 0x01
  - 0x69 || 0x55 -> 0x01

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  - 0x69 || 0x55 -> 0x01

Watch out for && vs. & (and || vs. |)…
a common bug in C programming

Representing Positive and Negative Integers

- Sign-Magnitude - MSB represents sign (0 for +ve, 1 for -ve)
- One's Complement of $x = 2^n - x - 1$ (complement individual bits)

Problem: Balanced representation, but two values for 0

Solution:

- Two's Complement of $x = 2^n - x$ (radix complement; most common representation)
  - single bit pattern for 0
  - ensures that $x + (-x)$ is 0
  - still keeps 1 in MSB for a -ve number (sign bit)
  - 100... represents the most -ve number
  - E.g. 4-bit 2's complement number $1100_2 = -1x2^3 + 1x2^2 + 0x2^1 + 0x2^0 = -4_{10}$

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Integer Arithmetic

- Normal base 2 2's complement addition works on both positive and negative numbers
- Shortcuts
  - 2's complement = 1s' complement + 1
  - 2's complement representation of n digit number as n+m digit number --- sign extend

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  - **Conversion, casting**
  - Expanding, truncating
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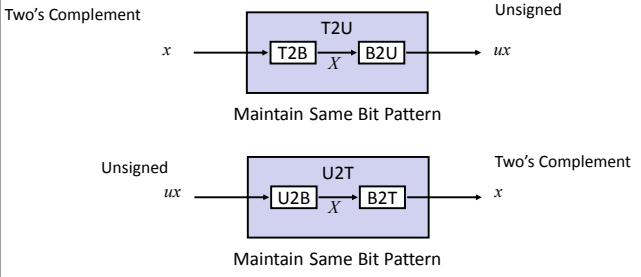
Mapping Between Signed & Unsigned



- Mappings between unsigned and two's complement numbers:
  Keep bit representations and reinterpret

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Mapping Signed ↔ Unsigned

Bits	Signed	Unsigned
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	-8	8
1001	-7	9
1010	-6	10
1011	-5	11
1100	-4	12
1101	-3	13
1110	-2	14
1111	-1	15

T2U

U2T

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Mapping Signed ↔ Unsigned

Bits	Signed	Unsigned
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	-8	8
1001	-7	9
1010	-6	10
1011	-5	11
1100	-4	12
1101	-3	13
1110	-2	14
1111	-1	15

=

+/- 16

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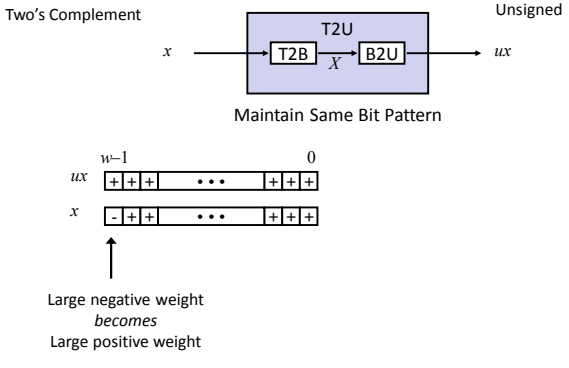
Relation between Signed & Unsigned



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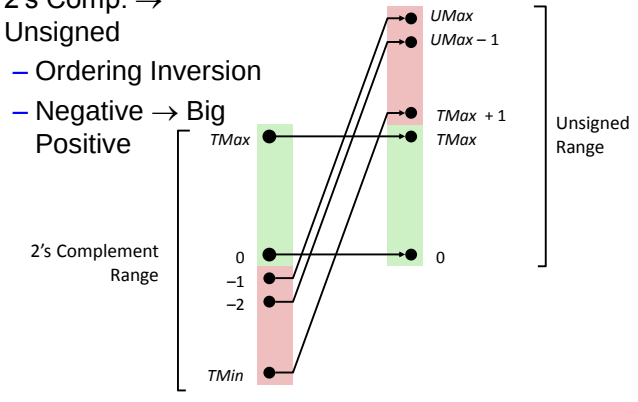
Conversion Visualized

- 2's Comp. → Unsigned
  - Ordering Inversion
  - Negative → Big Positive



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Signed vs. Unsigned in C

- Constants
  - By default are considered to be signed integers
  - Unsigned if have "U" as suffix

```
0U, 4294967259U
```

- Casting
  - Explicit casting between signed & unsigned same as U2T and T2U

```
int tx, ty;
unsigned ux, uy;
tx = (int) ux;
uy = (unsigned) ty;
```

  - Implicit casting also occurs via assignments and

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Casting Surprises

- Expression Evaluation
  - If there is a mix of unsigned and signed in single expression, ***signed values implicitly cast to unsigned***
  - Including comparison operations **<, >, ==, <=, >=**
  - Examples for *W* = 32: **TMIN = -2,147,483,648 , TMAX = 2,147,483,647**

Constant$_1$	Constant$_2$	Relation	Evaluation
0	0U		
-1	0		
-1	0U	==	unsigned
2147483647	-2147483647-1	<	signed
2147483647U	-2147483647-1	>	unsigned
-1	-2	>	signed
(unsigned)-1	-2	<	unsigned
2147483647	2147483648U	>	signed
2147483647	(int) 2147483648U	>	unsigned
		<	unsigned
		>	signed

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Summary
Casting Signed ↔ Unsigned: Basic Rules

- Bit pattern is maintained
- But reinterpreted
- Can have unexpected effects: adding or subtracting 2^w
- Expression containing signed and unsigned int
  - `int` is cast to `unsigned`!!

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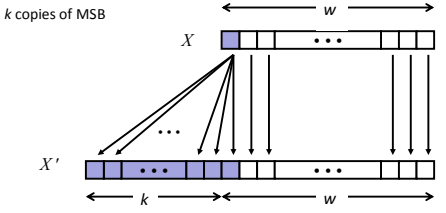
Today: Bits, Bytes, and Integers

- Representing information as bits
- Bit-level manipulations
- Integers
  - Representation: unsigned and signed
  - Conversion, casting
  - **Expanding, truncating**
  - Addition, negation, multiplication, shifting
  - Summary
- Representations in memory, pointers, strings

Sign Extension

- Task:
  - Given w-bit signed integer x
  - Convert it to $w+k$-bit integer with same value
- Rule:
  - Make k copies of sign bit:
  - $X' = \underbrace{x_{w-1}, \ldots, x_{w-1}}_{k \text{ copies of MSB}}, x_{w-1}, x_{w-2}, \ldots, x_0$



Sign Extension Example

```
short int x =  15213;
int      ix = (int) x;
short int y = -15213;
int      iy = (int) y;
```

	Decimal	Hex	Binary
x	15213	3B 6D	00111011 01101101
ix	15213	00 00 3B 6D	00000000 00000000 00111011 01101101
y	-15213	C4 93	11000100 10010011
iy	-15213	FF FF C4 93	11111111 11111111 11000100 10010011

- Converting from smaller to larger integer data type
- C automatically performs sign extension

Summary: Expanding, Truncating: Basic Rules

- Expanding (e.g., short int to int)
  - Unsigned: zeros added
  - Signed: sign extension
  - Both yield expected result
- Truncating (e.g., unsigned to unsigned short)
  - Unsigned/signed: bits are truncated
  - Result reinterpreted
  - Unsigned: mod operation
  - Signed: similar to mod
  - For small numbers yields expected behavior

Today: Bits, Bytes, and Integers

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- Bit-level manipulations
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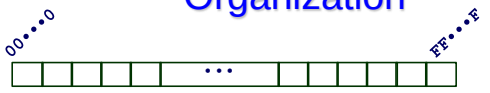
Byte-Oriented Memory Organization



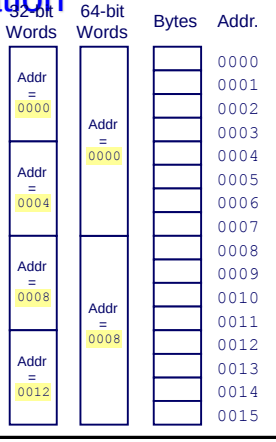
- Programs refer to data by address
  - Conceptually, envision it as a very large array of bytes
    - In reality, it's not, but can think of it that way
  - An address is like an index into that array
    - and, a pointer variable stores an address
- Note: system provides private address spaces to each "process"
  - Think of a process as a program being executed
  - So, a program can clobber its own data, but not that of others

Machine Words

- Any given computer has a "Word Size"
  - Nominal size of integer-valued data
    - and of addresses
  - Until recently, most machines used 32 bits (4 bytes) as word size
    - Limits addresses to 4GB (2^{32} bytes)
  - Increasingly, machines have 64-bit word size
    - Potentially, could have 18 EB (exabytes) of addressable memory
    - That's 18.4 X 10^{18}
  - Machines still support multiple data formats
    - Fractions or multiples of word size
    - Always integral number of bytes

Word-Oriented Memory Organization

- Addresses Specify Byte Locations
  - Address of first byte in word
  - Addresses of successive words differ by 4 (32-bit) or 8 (64-bit)



Example Data Representations

C Data Type	Typical 32-bit	Typical 64-bit	x86-64
`char`	1	1	1
`short`	2	2	2
`int`	4	4	4
`long`	4	8	8
`float`	4	4	4
`double`	8	8	8
`long double`	–	–	10/16
pointer	4	8	8

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Byte Ordering

- So, how are the bytes within a multi-byte word ordered in memory?
- Conventions
  - Big Endian: Sun, PPC Mac, Internet
    - Least significant byte has highest address
  - Little Endian: x86, ARM processors running Android, iOS, and Windows
    - Least significant byte has lowest address

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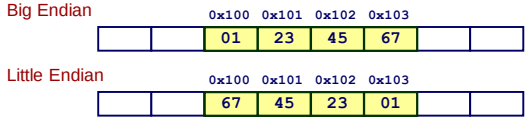
Byte Ordering Example

- Example
  - Variable x has 4-byte value of 0x01234567
  - Address given by &x is 0x100



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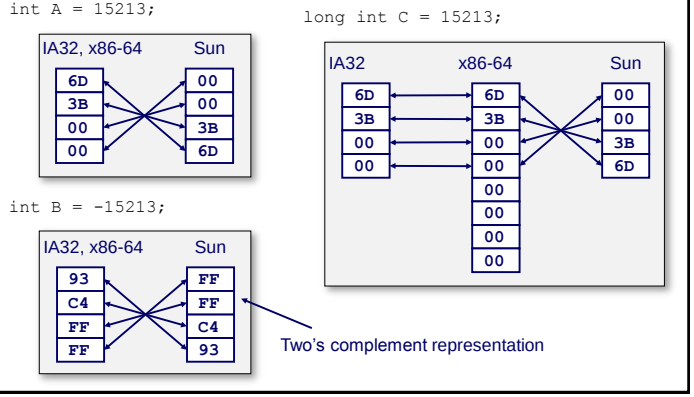
Representing Integers

Decimal: 15213
Binary: 0011 1011 0110 1101
Hex: 3 B 6 D



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Examining Data Representations

- Code to Print Byte Representation of Data
  - Casting pointer to unsigned char * allows treatment as a byte array

```
typedef unsigned char *pointer;

void show_bytes(pointer start, size_t len){
  size_t i;
  for (i = 0; i < len; i++)
    printf("%p\t0x%.2x\n",start+i, start[i]);
  printf("\n");
}
```

Printf directives:
%p: Print pointer
%x: Print Hexadecimal

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`show_bytes` Execution Example

```
int a = 15213;
printf("int a = 15213;\n");
show_bytes((pointer) &a, sizeof(int));
```

Result (Linux x86-64):

```
int a = 15213;
0x7fffb7f71dbc    6d
0x7fffb7f71dbd    3b
0x7fffb7f71dbe    00
0x7fffb7f71dbf    00
```

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Representing Pointers

```
int B = -15213;
int *P = &B;
```

Sun	IA32	x86-64
EF	AC	3C
FF	28	1B
FB	F5	FE
2C	FF	82
		FD
		7F
		00
		00

Different compilers & machines assign different locations to objects

Even get different results each time run program

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Representing Strings

```
char S[6] = "18213";
```

- Strings in C
  - Represented by array of characters
  - Each character encoded in ASCII format
    - Standard 7-bit encoding of character set
    - Character "0" has code 0x30
      - Digit *i* has code 0x30+*i*
  - String should be null-terminated
    - Final character = 0
- Compatibility
  - Byte ordering not an issue

IA32	Sun
31	31
38	38
32	32
31	31
33	33
00	00

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Integer C Puzzles

Initialization

```
int x = foo();
int y = bar();
unsigned ux = x;
unsigned uy = y;
```

- `x < 0` □□ `((x*2) < 0)`
- `ux >= 0`
- `x & 7 == 7` □□ `(x<<30) < 0`
- `ux > -1`
- `x > y` □□ `-x < -y`
- `x * x >= 0`
- `x > 0 && y > 0` □□ `x + y > 0`
- `x >= 0` □□ `-x <= 0`
- `x <= 0` □□ `-x >= 0`
- `(x|-x)>>31 == -1`
- `ux >> 3 == ux/8`
- `x >> 3 == x/8`
- `x & (x-1) != 0`

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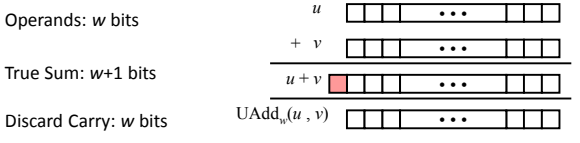
Unsigned Addition



- Standard Addition Function
  - Ignores carry output
- Implements Modular Arithmetic

  $s = \mathrm{UAdd}_w(u , v) = u + v \mod 2^w$

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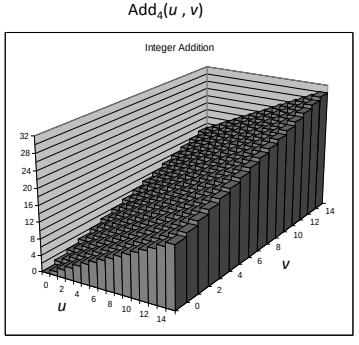
Visualizing (Mathematical) Integer Addition

- Integer Addition
  - 4-bit integers u, v
  - Compute true sum $\text{Add}_4(u , v)$
  - Values increase linearly with u and v
  - Forms planar surface



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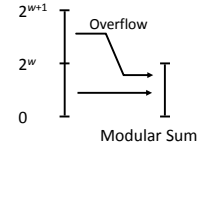
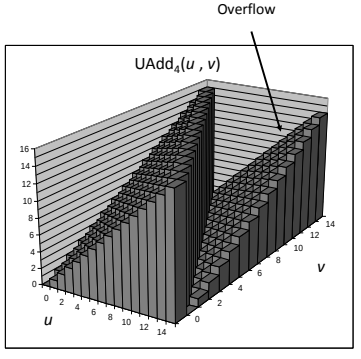
Visualizing Unsigned Addition

- Wraps Around
  - If true sum ≥ 2^w
  - At most once





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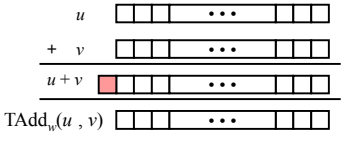
Two's Complement Addition

Operands: w bits

True Sum: w+1 bits

Discard Carry: w bits



- TAdd and UAdd have Identical Bit-Level Behavior
  - Signed vs. unsigned addition in C:

```
int s, t, u, v;
s = (int) ((unsigned) u + (unsigned) v);
t = u + v
```

  - Will give `s == t`

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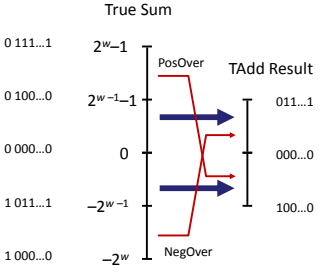
TAdd Overflow

- Functionality
  - True sum requires w+1 bits
  - Drop off MSB
  - Treat remaining bits as 2's comp. integer



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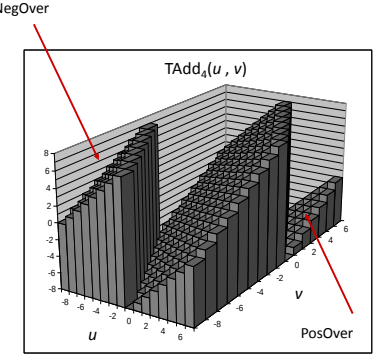
Visualizing 2's Complement Addition

- Values
  - 4-bit two's comp.
  - Range from -8 to +7
- Wraps Around
  - If sum ≥ 2^{w-1}
    - Becomes negative
    - At most once
  - If sum < -2^{w-1}
    - Becomes positive
    - At most once



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Exceptions

- Overflow: number too large to be represented in n bits
- Overflow condition for O = A+B: !MSBA.!MSBB.MSBO + MSBA.MSBB.!MSBO
- Detection of overflow language specific
  - ignored in C, required in Fortran
- Memory addressing arithmetic on unsigned numbers
- An exception/interrupt generated on overflow for signed arithmetic

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Multiplication

- Goal: Computing Product of w-bit numbers x, y
  - Either signed or unsigned
- But, exact results can be bigger than w bits
  - Unsigned: up to $2w$ bits
    - Result range: $0 \le x * y \le (2^w - 1)^2 = 2^{2w} - 2^{w+1} + 1$
  - Two's complement min (negative): Up to $2w$-1 bits
    - Result range: $x * y \ge (-2^{w-1})*(2^{w-1}-1) = -2^{2w-2} + 2^{w-1}$
  - Two's complement max (positive): Up to $2w$ bits, but only for $(TMin_w)^2$
    - Result range: $x * y \le (-2^{w-1})^2 = 2^{2w-2}$
- So, maintaining exact results…
  - would need to keep expanding word size with each product computed
  - is done in software, if needed
    - e.g., by "arbitrary precision" arithmetic packages

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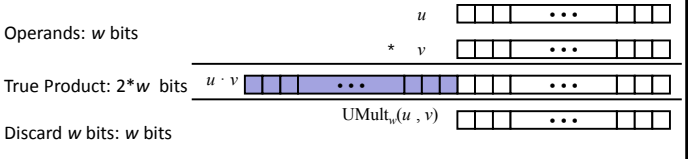
Unsigned Multiplication in C



- Standard Multiplication Function
  - Ignores high order w bits
- Implements Modular Arithmetic

  $\text{UMult}_w(u , v) = u \cdot v \mod 2^w$

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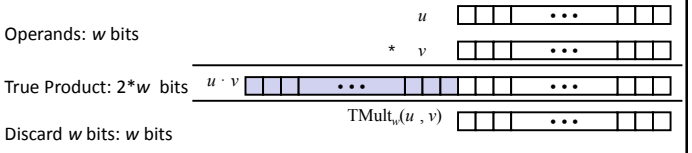
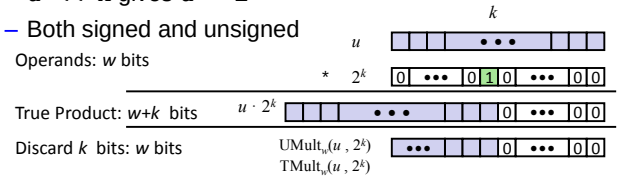
Signed Multiplication in C



- Standard Multiplication Function
  - Ignores high order *w* bits
  - Some of which are different for signed vs. unsigned multiplication
  - Lower bits are the same

Power-of-2 Multiply with Shift

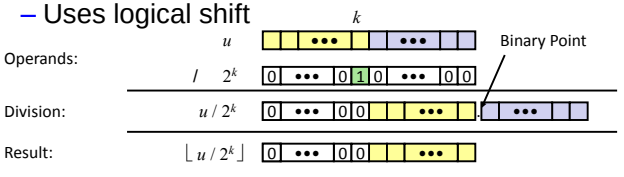
- Operation
  - `u << k` gives `u * 2^k`
  - Both signed and unsigned



- Examples
  - `u << 3 == u * 8`
  - `(u << 5) - (u << 3) == u * 24`
  - Most machines shift and add faster than multiply
    - Compiler generates this code automatically

Unsigned Power-of-2 Divide with Shift

- Quotient of Unsigned by Power of 2
  - `u >> k` gives $\lfloor$ `u / 2^k` $\rfloor$
  - Uses logical shift



	Division	Computed	Hex	Binary
x	15213	15213	3B 6D	00111011 01101101
x >> 1	7606.5	7606	1D B6	00011101 10110110
x >> 4	950.8125	950	03 B6	00000011 10110110
x >> 8	59.4257813	59	00 3B	00000000 00111011

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- Representing information as bits
- Bit-level manipulations
- Integers
  - Representation: unsigned and signed
  - Conversion, casting
  - Expanding, truncating
  - Addition, negation, multiplication, shifting
  - **Summary**
- Representations in memory, pointers, strings

Arithmetic: Basic Rules

- Addition:
  - Unsigned/signed: Normal addition followed by truncate, same operation on bit level
  - Unsigned: addition mod 2^w
    - Mathematical addition + possible subtraction of 2^w
  - Signed: modified addition mod 2^w (result in proper range)
    - Mathematical addition + possible addition or subtraction of 2^w
- Multiplication:
  - Unsigned/signed: Normal multiplication followed by truncate, same operation on bit level
  - Unsigned: multiplication mod 2^w
  - Signed: modified multiplication mod 2^w (result in proper range)

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When Should I Use Unsigned?

- *Don't* use without understanding implications
  - Easy to make mistakes

```
unsigned i;
for (i = cnt-2; i >= 0; i--)
  a[i] += a[i+1];
```

  - Can be very subtle

```
#define DELTA sizeof(int)
int i;
for (i = CNT; i-DELTA >= 0; i-= DELTA)
  . . .
```

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Counting Down with Unsigned

- Proper way to use unsigned as loop index

```
unsigned i;
for (i = cnt-2; i < cnt; i--)
  a[i] += a[i+1];
```

- See Robert Seacord, *Secure Coding in C and C++*
  - C Standard guarantees that unsigned addition will behave like modular arithmetic
    - 0 – 1 → *UMax*
- Even better

```
size_t i;
for (i = cnt-2; i < cnt; i--)
  a[i] += a[i+1];
```

  - Data type `size_t` defined as unsigned value with length = word size
  - Code will work even if `cnt` = *UMax*
  - What if `cnt` is signed and < 0?

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Why Should I Use Unsigned? (cont.)

- *Do* Use When Performing Modular Arithmetic
  - Multiprecision arithmetic
- *Do* Use When Using Bits to Represent Sets
  - Logical right shift, no sign extension

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