

Problem 5 *Another NP-complete graph problem*

Note: a weighted graph (for our purposes) is one whose edges are labelled with nonnegative integer weights. A simple path (or cycle) is one that doesn't intersect itself. The *cost* of a path (or cycle) is the sum of weights of the edges traversed.

WEIGHTED-CYCLE =_{def} $\{\langle G, k \rangle \mid G \text{ is an undirected weighted graph that contains a simple cycle of cost } k\}$.

Note: This problem is of interest for example in designing a road race that needs to be of a particular length, has no overlapping or self-intersecting segments, and starts and finishes at the same cross-roads.

Show that WEIGHTED-CYCLE is NP-complete, using a reduction from SUBSET-SUM to WEIGHTED-CYCLE. *Hint:* One approach is to think in terms of loops that branch outward from a central region, like petals of a flower. But there are other equally good approaches.

Problem 6 Show that SAT-SAT =_{def} $\{\langle \phi \rangle \mid \phi \text{ is a boolean formula with 2 or more satisfying assignments}\}$ is NP-complete.

Problem 7 Let INTEGRAL-ROOTS be the problem of deciding whether an arbitrary polynomial with integer coefficients has integral roots, i.e., whether we can choose integer values of the variables for which the polynomial is 0. (For example, $x^2y - 2xy - 2x^2 + 4x + y - 2$ is a positive instance of this problem, with roots $x = 1, y = 2$; $xyz^2 - xz^2 - yz^2 - 4xy + z^2 + 4x + 4y - 4$ is another positive instance, with roots $x = 1, y = 1, z = 2$.)

Show that every problem in NP is polynomial-time reducible to INTEGRAL-ROOTS. Does this show that INTEGRAL-ROOTS is NP-complete? Why or why not?