Lecture 1: Overview & Fundamental Concepts

What is a Problem?

Classification (recognition) problem: Determining for a class of inputs whether a given input belongs to a certain class;

e.g., recognizing strings containing a specified substring;
recognizing well-formed arithmetic expressions;
recognizing prime numbers;
recognizing satisfiable boolean formulas; etc.

Mapping problem: Mapping each of a class of inputs to an ouput related to the input in some specified way;

e.g., Making up/down/wait decisions for 1 or more elevators,
as a function of buttons pushed on various floors;
mapping character strings to phonemes for speech generation;
finding a solution to a boolean formula; etc.

Specifying a Problem Declaratively (Descriptively)

We often use **ordinary language plus mathematical notation** to specify a problem declaratively;

e.g., "binary strings containing 0011";
$\{$x0011y $|$ x, y are binary strings$\}$;
$\{\langle G, k\rangle |\ k$ is the size of the largest clique of G,
where G is an undirected graph$\}$; etc.

We may also use various **declarative formalisms** for problem description, such as regular expressions, formal grammars, or the λ-calculus;

e.g., $(\mathbf{0} \cup \mathbf{1})^*\mathbf{0011}(\mathbf{0} \cup \mathbf{1})^*$;
$S \rightarrow 0011 \mid BS \mid SB,\ \ B \rightarrow 0 \mid 1$;
$\lambda xy(x+y)$; etc.

What Is Computation?

Computation is a systematic way of obtaining an answer to a problem.

The systematic nature of computation allows the use of **computing devices** for actual computation.

The abstraction of devices of the “same kind” is a **model**

- “Is this model ‘different’ from that model?”
- “What problems can be solved under this model?”

Problem Classes

From a declarative perspective:
Class of Problems = Descriptive Formalism + Interpretation of the Formalism; (or, an informal description of the class)

From a computational perspective:
Class of Problems = Computation model + Concept of “Solving a Problem”

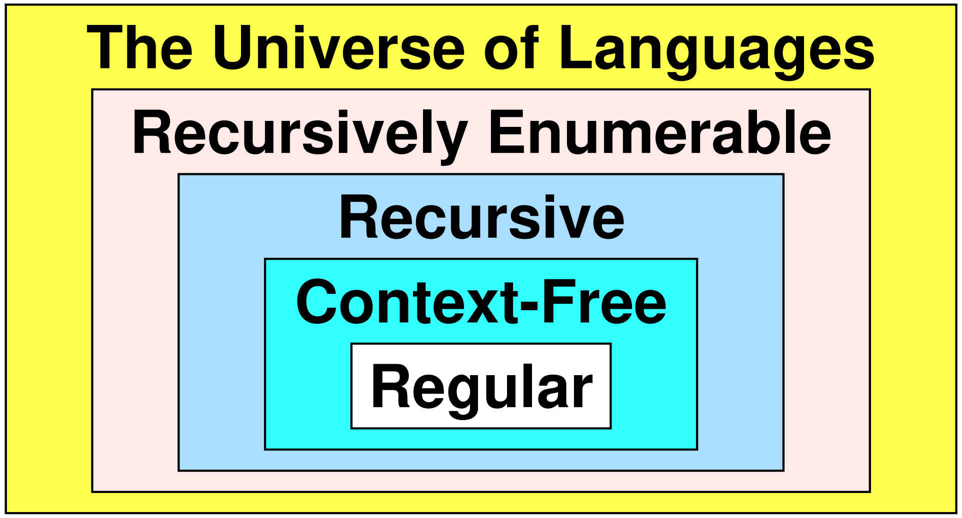
- “Are this model (or descriptive formalism) and that model (or descriptive formalism) equivalent?” $\longrightarrow$ “Is class A equal to class B?”
- “Can any problem be solved under this model (or described in this formalism)?” $\longrightarrow$ “Exactly what is class A?”

Computation Resources

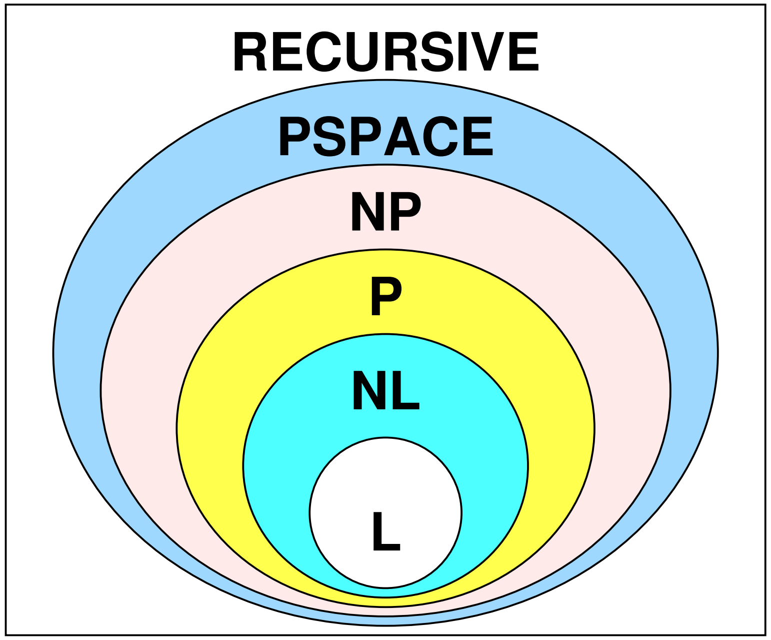
Distinction between resource-bounded computation and resource-unbounded computation.

How does bounding resources affect the power of computation?

Class Overview: Resource-Unbounded



Class Overview: Resource-Bounded



Alphabet, Strings, Languages, etc.

See page 13 of the textbook

Alphabet, Strings, Languages, etc.

- An **alphabet** is any finite set, whose members are called **symbols**.
- A **string (or word) over an alphabet** is a sequence of symbols from the alphabet written one after another.
- The **length** of a word w, denoted by $|w|$, is the number of symbols in it.
- The **empty string**, denoted by ϵ, is the string with no symbols in it.

Alphabet, Strings, Languages, etc. (cont'd)

- A string z is a **substring** of w if z appears consecutively within w.
- The **concatenation** of strings x and y is the string constructed by appending y after x.
- A **language** is a collection of strings.
- The **complement** of a language is the collection of all non-members.
- A **class** is a collection of languages.

Alphabet, Strings, Languages, etc. (cont'd)

Example Let $\Sigma = \{0, 1\}$ be an alphabet. The symbols of Σ are 0 and 1.

Let $z = 001111010$. Then z is a string over Σ. 1111 is a substring of z while 11111 is not. The concatenation of $a = 000$ and $b = 111$ is 000111.

The set of all strings over Σ with the same number of 0s and 1s is a language. The collection of all languages over Σ is a class.

Boolean Logic

A **Boolean variable** takes on one of 0 (FALSE) and 1 (TRUE). The **negation** of x, denoted by $\overline{x}$ or $\neg x$, is $1 - x$.

We will be using six binary Boolean operators:

(x, y)	$(0, 0)$	$(0, 1)$	$(1, 0)$	$(1, 1)$
$\wedge$	0	0	0	1
$\vee$	0	1	1	1
$\rightarrow$	1	1	0	1
$\leftarrow$	1	0	1	1
$\leftrightarrow$	1	0	0	1
$\oplus$	0	1	1	0

Boolean Logic (cont'd)

A **predicate** is a **function** whose **range** is TRUE, FALSE. A **relation** is a predicate whose number of arguments is a fixed constant.

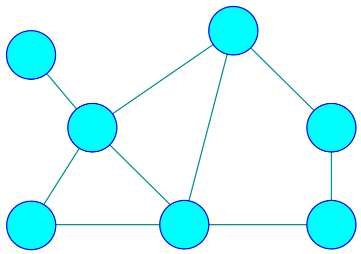
Properties of binary relation R over domain D:

- **reflexive** for all $x \in D$, xRx
- **symmetric** for all $x, y \in D$, $xRy \leftrightarrow yRx$
- **transitive** for all $x, y, z \in D$, $xRy \wedge yRz \rightarrow xRz$

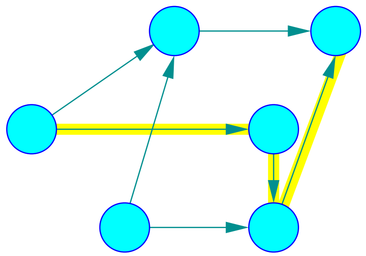
An **equivalence relation** is a binary relation that is reflexive, symmetric, and transitive

Graphs

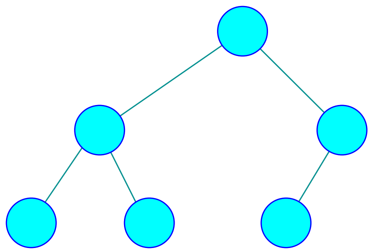
A **graph** consists of **nodes (vertices)** and **edges**. A **path** is a sequence of edges (or a sequence of nodes) that connects two nodes. A **tree** is a **connected, undirected graph** without **simple cycles**.



an undirected graph



a directed graph



a tree