

### Homework problems:

**12.1 (due Nov 30, 2006)** There are  $n$  bottles which contain different mixtures of three chemicals called  $A, B, C$ . The  $i$ -th bottle contains the chemicals in ratio  $a_i : b_i : c_i$  (thus,  $a_i/(a_i + b_i + c_i)$  fraction of the  $i$ -th bottle is chemical  $A$ ,  $b_i/(a_i + b_i + c_i)$  fraction of the  $i$ -th bottle is chemical  $B$ , and  $c_i/(a_i + b_i + c_i)$  fraction of the  $i$ -th bottle is chemical  $C$ ). We want to know whether it is possible to obtain a mixture containing the chemicals  $A, B, C$  in ratio  $a : b : c$  by mixing various amounts from the bottles. Give an efficient algorithm for this problem.

For example, if the input is  $n = 2$ , the ratios in the bottles are  $1 : 1 : 2$  and  $3 : 3 : 1$ , and we want to obtain mixture with ratio  $1 : 1 : 1$  then the answer is YES (we can take 2 parts from the first bottle and 1 part from the second bottle).

**12.2 (due Nov 30, 2006)** Solve the following linear program:

$$\begin{aligned} \max \quad & \sum_{i=1}^9 x_i, \\ & x_1 + x_2 \leq 1, \\ & x_2 + x_3 \leq 1, \\ & x_3 + x_4 \leq 1, \\ & x_4 + x_5 \leq 1, \\ & x_5 + x_6 \leq 1, \\ & x_6 + x_7 \leq 1, \\ & x_7 + x_8 \leq 1, \\ & x_8 + x_9 \leq 1, \\ & x_9 + x_1 \leq 1, \\ & x_i \geq 0, i = 1, \dots, 9. \end{aligned}$$

You can (but don't have to) use linear programming solvers (such as, `lpsolve`).

**12.3 (due Nov 30, 2006)** Construct the linear program dual to the following linear program:

$$\begin{aligned} \max \quad & x_1 + 2x_3 + 3x_3 + 4x_4 \\ & x_2 + x_3 + x_4 \leq 1 \\ & x_1 + x_3 + x_4 \leq 2 \\ & x_1 + x_2 + x_4 \leq 3 \\ & x_1 + x_2 + x_3 \leq 4 \\ & x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0. \end{aligned}$$