

1 Schedule

Problem sessions:

Wed, Dec. 5, 8:45pm-9:45pm

Mon, Dec. 10, 7pm-8pm

Wed, Dec. 12, 8:45pm-9:45pm

The quiz will be on Thursday, Dec. 13.

2 List of algorithms covered in the class

(B-basic, I-intermediate, A-advanced):

I: Vertex cover (p. 278, DSV).

I: Clustering (p. 280, DSV).

I: TSP (p. 282, DSV).

A: Knapsack (p. 283, DSV).

A: Set-cover.

3 Basic material

Important concepts, problems, theorems, and algorithms:

- NP, NP-hard, NP-complete, reduction.
- Basic NP-complete problems (SAT, 3SAT, vertex cover, clique, k -coloring, integer linear programming)

Testing method:

- Solve a small instance of the basic NP-complete problems.
- Give a (simple) reduction between a pair basic NP-complete problems.

Example problems (homework):

8.1 (due Dec 6, 2007) Let $V = \{0, 1, 2, 3\} \times \{0, 1, 2, 3\}$ and $E = \{(a, b), (c, d)\} \in \binom{V}{2} \mid a = c \text{ or } b = d\}$. Find the largest clique in $G = (V, E)$.

8.2 (due Dec 6, 2007) Let $V = \{0, 1, 2, 3\} \times \{0, 1, 2, 3\}$ and $E = \{(a, b), (c, d)\} \in \binom{V}{2} \mid a = c \text{ or } b = d\}$. Find the smallest vertex cover in $G = (V, E)$.

8.3 (due Dec 6, 2007) Give a reduction from 3-SAT to integer linear programming.

4 Additional homework

8.4 (due Dec 11, 2007) [Problem rating: I.] A vertex cover of a graph $G = (V, E)$ is a subset S of vertices such that for each edge $e \in E$ at least one of its endpoints is in S . Consider the following optimization problem:

VERTEX-COVER

INSTANCE: A graph G .

SOLUTION: A vertex cover S of size G .

OBJECTIVE: Minimize the size of S .

Consider the following reduction from VERTEX-COVER to INTEGER LINEAR PROGRAMMING. For each vertex $i \in V = \{1, \dots, n\}$ we will have a variable x_i and constraints $0 \leq x_i$ and $x_i \leq 1$. For each edge $\{i, j\} \in E$ we will have a constraint $x_i + x_j \geq 1$. Finally, the objective is to minimize $x_1 + \dots + x_n$. Thus our integer linear program is

$$\begin{aligned} \min \quad & \sum_{i \in V} x_i \\ & x_i \leq 1, \text{ for } i \in V, \\ & x_i \geq 0, \text{ for } i \in V, \\ & x_i + x_j \geq 1, \text{ for } \{i, j\} \in E, \\ & x_i \in \mathbb{Z}, \text{ for } i \in V \quad (\text{integrality}). \end{aligned} \tag{1}$$

Let O be the optimum of (1).

Now view (1) as a linear program (i.e., drop the integrality constraint). Let R be the optimum of this linear program.

a) Which of the following two is always true?

$$R \leq O \qquad O \leq R$$

b) Find a graph G for which the values of O and R are different.

8.5 (due Dec 11, 2007) [Problem rating: A.] Prove that if the graph G in Problem 8.4 is bipartite, then $O = R$.

5 Additional problems from the book (do not turn in)

Try to solve the following problems. A few of them will be on the quiz. We will go over the ones that you choose in the problem sessions.

- 8.1, 8.3, 8.4, 8.6, 8.14, 8.18, 9.7, 9.9.