

Linear programming (part 5 of CSC 282),

<http://www.cs.rochester.edu/~stefanko/Teaching/11CS282>

1 Schedule

"Homework" problem session is in CSB 601, 6:15-7:15pm on Nov. 30 (Wednesday); held by Rintaro Kuroiwa and Josh Pawlicki.

Homework is **due Dec 1** (Thursday).

"Exam" problem session is in CSB 601, 4:45-5:45pm on Dec. 5 (Monday).

EXAM #5 will be on **Tuesday, Dec. 6**.

Concepts and methods that will be tested on the exam:

- formulating a linear program for a problem (similar to problems 5.4, 5.5, 5.6, 5.7, and 5.8 below),
- duality (similar to problem 5.3 below),
- zero-sum games (similar to problems 5.1 and 5.2 below).

2 Basic Homework - solve and turn in

5.1 (due Dec 1) Compute the value of the zero-sum two person game given by the following payoff matrix:

$$\begin{pmatrix} 2 & 4 \\ 1 & 3 \\ 4 & 2 \end{pmatrix}$$

5.2 (due Dec 1) Consider the zero-sum game with the following pay-off matrix

$$\begin{pmatrix} x & -3 \\ -2 & 4 \end{pmatrix}.$$

For what value of x is the value of the game 0? Clearly explain how you obtained the answer.

5.3 (due Dec 1) Construct the linear program dual to the following linear program:

$$\begin{aligned} \max x_1 + 2x_2 + 3x_3 + 4x_4 \\ x_2 + x_3 + x_4 &\leq 1 \\ x_1 + x_3 + x_4 &\leq 2 \\ x_1 + x_2 + x_4 &\leq 3 \\ x_1 + x_2 + x_3 &\leq 4 \\ x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0. \end{aligned}$$

Find the optimal solution of the primal and the dual problem. **Use a linear programming solver** to obtain the solutions (for example you can use freeware `lpsolve` or function `Maximize` in MATHEMATICA (installed in some labs)).

5.4 (due Dec 1) Krusty the Clown purchased a new super-efficient SUV with two engines. The first engine runs on a 2:1 mixture of gasoline and ethanol and gets 12 mpg. The second engine runs on a 1:2 mixture of gasoline and ethanol and gets 22 mpg. Krusty has 12 gallons of gasoline and 14 gallons of ethanol. How far can he get?

(a) Write a linear program for the problem. You are allowed to have only two variables in your linear program.

(b) Solve the linear program (you do not need to use the simplex method).

(c) Write down the dual of your linear program from part (a).

(d) Solve the dual program (you do not need to use the simplex method).

5.5 (due Dec 1) There are n bottles which contain different mixtures of three chemicals called A, B, C . The i -th bottle contains the chemicals in ratio $a_i : b_i : c_i$ (thus, $a_i/(a_i + b_i + c_i)$ fraction of the i -th bottle is chemical A , $b_i/(a_i + b_i + c_i)$ fraction of the i -th bottle is chemical B , and $c_i/(a_i + b_i + c_i)$ fraction of the i -th bottle is chemical C). We want to know whether it is possible to obtain a mixture containing the chemicals A, B, C in ratio $a : b : c$ by mixing various amounts from the bottles. Give an efficient algorithm for this problem.

For example, if the input is $n = 2$, the ratios in the bottles are $1 : 1 : 2$ and $3 : 3 : 1$, and we want to obtain mixture with ratio $1 : 1 : 1$ then the answer is YES (we can take 2 parts from the first bottle and 1 part from the second bottle).

3 Advanced Homework - solve and turn in

5.6 (due Dec 1) We are given n pairs of real numbers $(x_1, y_1), \dots, (x_n, y_n)$. We want to find real numbers a, b which minimize

$$\max_{i \in \{1, \dots, n\}} |ax_i + y_i - b|.$$

Write a linear program for this problem.

5.7 (due Dec 1) We are given n pairs of real numbers $(x_1, y_1), \dots, (x_n, y_n)$. We want to find real numbers a, b which minimize

$$\sum_{i \in \{1, \dots, n\}} |ax_i + y_i - b|.$$

Write a linear program for this problem.

5.8 (due Dec 1) We would like to find the largest axis-aligned¹ square inscribed in a convex polygon P . The polygon P is given as a intersection of n half-planes, that is, we are given $a_1, b_1, c_1, \dots, a_n, b_n, c_n$ which define P to be the set of points (x, y) such that $a_i x + b_i y \leq c_i$ for all $i \in \{1, \dots, n\}$. Give a linear program for the problem. Clearly explain the meaning of the variables and the constraints in your linear program.

¹a square is axis-aligned if its sides are parallel with the x and y axis

5.9 (due Dec 1) We are given n pairs of real numbers $(x_1, y_1), \dots, (x_n, y_n)$. We want to find real numbers a, b which minimize

$$\sum_{i \in \{1, \dots, n\}} (ax_i + y_i - b)^2.$$

Find a formula for a and b (you do NOT need linear programming for this one).

4 Additional problems from the book (do not turn in)

Try to solve the following problems. A few of them will be on the quiz. We will go over the ones that you choose in the problem sessions.

- 7.1, 7.2, 7.3, 7.4, 7.6, 7.7, 7.8, 7.9, 7.11, 7.12, 7.13, 7.14, 7.15, 7.19, 7.27, 7.28.