CSC 446 Notes: Sparse Coding

We being with equation 7 of Lee, Battle, Raina and Ng (2006):

$$
\begin{aligned}
L(B,\lambda) &= \mathrm{Tr}((X-BS)^T(X-BS)) + \sum_j \lambda_j \left(\sum_i B_{ij}^2 - c \right) \\
&= \mathrm{Tr}(X^TX - 2X^TBS + S^TB^TBS + \Lambda B^TB - c\Lambda) \\
&= \mathrm{Tr}(X^TX - 2BSX^T + (SS^T + \Lambda)B^TB - c\Lambda) \\
\frac{\partial L}{\partial (B^T)} = 0 &= -2SX^T + 2(SS^T + \Lambda)B^T \\
B^T &= (SS^T + \Lambda)^{-1}(SX^T) \\
g(\lambda) &= \mathrm{Tr}(XX^T - 2XS^T(SS^T + \Lambda)^{-1}SX^T + SX^T(SS^T + \Lambda)SX^T - c\Lambda) \\
&= \mathrm{Tr}(XX^T - XS^T(SS^T + \Lambda)^{-1}SX^T - c\Lambda)
\end{aligned}
$$

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