Parallel Programming

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A Sidetrack

- Matrix multiplication

![Matrix multiplication diagram]

- Efficient use of cache
- Block-by-block multiplication

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Gaussian Elimination: Dependencies

- Simplification - ignore pivoting and final solving step
- Reduce an equation matrix into an equivalent upper-diagonal

![Gaussian Elimination diagram 1]

- dependencies:
  - Outer loop instances (e.g., \( i=2 \) depends on \( i=1 \))?
  - Inner loop instances (e.g., \( j=3 \) depends on \( j=2 \))?
  - Within one inner loop instance?

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Gaussian Elimination: Task Decomposition

- Parallelism:
  - Inner loop instances (row-wise)
  - Within one inner loop instance (column-wise)

- Task decomposition:
  - Row, column, 2-dimensional

![Gaussian Elimination diagram 2]
Gaussian Elimination: Partitioning Granularity

- Row partitioning at different granularities
  - Row by row, cyclically
  - Every proc gets a contiguous chunk
  - Every proc gets a block of rows

- A good block size to efficiently utilize processor cache

Comparison of Alternative Approaches

- Row partitioning, column partitioning, 2-dimensional partitioning
- Blocking at different sizes
- Static vs. dynamic task assignment
- ... ...

Suggestions on Testing

- Incremental testing:
  - Try an artificial, small, dense input matrix first
  - Try to parallelize without partial pivoting first

Irregular Parallelism

- Real problems contain large, sparse matrices
- Solve them as dense matrices waste time on zero-element operations
- Sparse matrix computation
  - Load imbalance
  - Managing nonzero fillins
Example Speedup Results

- The speed ratio over the best sequential run

![Graphs showing speedup results with number of processors on the x-axis and speedup on the y-axis.]

Alternative Methods for Solving Linear Equations

- Faster solutions if we tolerate less precision
  - Less-than-precise pivoting
  - Iterative method

Instructional-Level Parallelism

- Instructional-level parallelism
  - Single CPU core, but multiple functional units (arithmetic computation, floating-point computation, ...)
  - Fetch/decode multiple instructions at a time and execute them in parallel
  - Possibly retire multiple instructions in a cycle
- Manage control dependencies
  - Stop before branch
  - Speculative execution on branch but rollback if mis-speculate
- Manage data dependencies
  - Recognize and observe dependencies
  - Resolve artificial dependencies through renaming

Cache Coherence Problem

- In shared memory multiprocessors (except multithreading), each processor has a local cache
  - After one processor updates the data, another processor's local copy may be incoherent
- For each data item in memory, additional copies may exist in processor local caches
  - After one processor updates the data, another processor's local copy may be incoherent
  - In cases of write-through and write-back caches
Coherence means the system semantics is the same as that of a system without multiple processor-local caches.

Multiprocessor cache coherent if there exists a hypothetical sequential order of all operations for each data location:
- returned value in the read operation is that written by last write in the sequential order
- the sequential order matches the order of operations from each processor

Sequential Memory Consistency
- It means the memory semantics is the same as that of a uni-processor system
- Sequential consistent if there exists a hypothetical sequential order of all operations on all locations:
  - returned value in the read operation is that written by last write in the sequential order
  - the sequential order matches the order of operations from each processor