In-Class Semi-Workshop-Like Problems for 2018/2/7

(last updated 2018/1/31/6:21pm)

CSC 280: Computer Models & Limitations
Spring 2018
Instructor: Lane A. Hemaspaandra
Grad TA: Andrew Read-McFarland

The group(s) on point for each problem are listed at its start. But if you have time, do look also at the other groups’ problems; they might be challenging and fun!

It would not be wildly shocking if these problems were related to some problems that might be on the an Exercise Set.

As usual, each group (or group pair for the last) should come into 2/5’s class with their answer on a sheet of paper ready to project and with someone on point to present their answer.

1 Cute Problem (... Well, Problem with a Cute, Smile-Inducing, Satisfying Solution)

1. [Thurs-late-Shir] (Warning: You’ll likely need to wait until after the 2/5 lecture—the coverage of Chapter 1 Part 3—to be able to solve this) \( L^r \) is the set of all strings whose reverses are in \( L \). For example, if \( L = \{1100, 1110\} \) then \( L^r = \{0011, 0111\} \). Prove that if \( L \) is regular, then so is \( L^r \).

Now, clearly one could do so by building an NFA that basically goes backward from a (guessed) final state to try to end up at the original start state.

But after seeing what you cover in the 2/5 lecture, please instead give a much more beautiful proof that the regular sets are closed under reversal—one that doesn’t require you to get down and hack 5-tuples and machines? (Hint: Think “structural induction” and regular expressions!) (Note: Since a lot of things will be gone over in class, you don’t have to give an utterly detailed proof, but please do try to see the key idea/approach.)

2 Moderately Easy Problems but on Relatively New-to-Us Tool

1. [Thurs-late-Brandon] Let our alphabet be \( \{a, b, c, d\} \). Let \( L = \{ab^nc^nd \mid n \geq 0\} \). Prove (using the pumping lemma) that \( L \) is not regular. (Be careful. Make sure to not forget the case(s) where \( y \) is a or contains a and some b's!)

2. [Thur-early] Consider trying to use the pumping lemma to show that the following language over the alphabet \( \{0, 1\} \) is not regular: \( L = \{w \mid w \text{ has the same number of 0's and 1's in it}\} \). (Yes, this one basically one of the examples from the slides.) Professor Foo tries to do this by a standard pumping lemma proof. So Foo lets \( p \geq 1 \) be an arbitrary natural number. And then if \( p \) is even lets \( w \in L \) be \((01)^{p/2}\) and if \( p \) is odd lets \( w \in L \) be \((01)^{\lfloor p/2 \rfloor}0 \). Will this lead to a successful proof? Or is the professor doomed, and if so, why? (Yes, if you were listening carefully you would have heard me touch in precisely this issue in class on while covering Chapter 1.4; but either way, this problem is still worth doing, as it makes the point that one’s choices, such as of \( w \), require great care.)
3 Harder Problem (But with Luck Easy for You by Now)

1. [Wed-late] Draw a DFA accepting the following language, over the alphabet \( \{0, 1, 2, 3, 4, 5, 6\} \):
   All strings that when interpreted as base-7 integers are multiples of 6 (i.e., are congruent, mod 6, to zero). (Ignore leading 0’s. Treat the empty string as if it is interpreted as the value 0. So 00000 represents 0 and 00011 represents the decimal number 8 and 101 represents the decimal number 50.)

4 Harder Problem (Outright Challenging if You Decide to Not Read the Hint and Do It Purely Yourself, Which Could Be Fun!)

1. [Wed-early-Brandon AND Wed-early-Shir, working together] (Warning: You’ll likely need to wait until after the 2/5 lecture—the coverage of Chapter 1 Part 3—to be able to solve this)
   You now know enough to prove that first thirds of regular sets are regular, and that 3rd 7ths of regular sets are regular, and lots more. Are there no limits to what we can do along the lines of such “fractional parts of regular languages” problems? Let us see!

   Let \( \Sigma = \{0, 1, 2\} \). Let \( A \) be an arbitrary regular set over alphabet \( \Sigma \). Consider the language of strings in \( A \) with the middle thirds cut out: \( L_A = \{ w \mid (\exists x, y, z)[|x| = |y| = |z| \land w = xz \land xyz \in A]\} \). Clearly that there exists a regular set \( A \) such that \( L_A \) is regular, e.g., if \( A = 0^* \) then \( L \) is \((00)^*\). Prove that there is a regular set \( A \) such at \( L_A \) is *not* regular. (Hint: Consider the regular set \( W = 0^*122^*10^* \). Cleverly choose a regular set to intersect with \( L_W \), so as to have the intersection be something that you can easily, using the pumping lemma, prove nonregular; and then do prove that intersection nonregular, and then draw your desired conclusion about \( L_W \). Yeah!)