A is some regular set

\[ L = \{ w \mid \exists w_1, w_2, w_4 \text{ such that } 4|w_1| = w_4, 2|w_1| = w_2, w_1 w_2 w_4 w_2 \in \mathcal{A} \} \]

States in NFA for \( L \) will be 14-tuples form will be
(guess start, guess end

\uparrow

1st 7th

\uparrow

2nd 7th etc

\hline

Start states =

\exists (q_{\text{start}}, q_i, q_i, q_i, \ldots \epsilon \mathcal{Q}) \mid q_f \in F,

(q_i, q_i, \ldots q_{i, k})
accepting: \[ \{(q_i, q_i, q_j, q_j, \ldots, q_f, q_f) \mid q_i, q_j, \ldots \in Q, q_f \in F\} \]

idea of transition

1st

2nd/3rd

4th/5th

11th

ending state

1st 7th guess ending of 1st 7th
start of 2nd 7th
\[
\delta'(C(q_0, q_6, \ldots, q_n), C) = \exists (q_a', q_b, q_c', q_d, \ldots, q_n) \quad \text{such that}
\]
\[
q_a' \in \delta(q_a, d), \text{ for some } d \in \Sigma,
\]
except for 5th, 7th, etc.

Explanation on next page

\[
Q' = \{(q_0, q_6, \ldots, q_n) \mid q_0, q_6, \ldots, q_n \in Q_3 \}
\]

∪ Σ (dummy start 3)
Assuming $j$ is the 9th letter of the alphabet...

$q_i' = s(q_i, c)$
$$\begin{align*}
\text{start states:} & \quad \{(q_i, q_i, q_j, q_j) \mid (q_i, q_i, q_j, q_j) \in \Sigma\} \\
\text{accepting states} & \quad \exists (q_i, q_i, q_j, q_k) \\
q_i, q_j, q_k \in \Sigma
\end{align*}$$

$$\exists y \in B \text{ such that } \delta(q_{\text{startA}}, yyy) = q_i$$
...

and

\[ \delta(q_k, y) = q_f \]

for some \( q_f \in F \)
If \( L \) is regular, then \( \exists p \) such that
\[ \forall w \in L, |w| \geq p, \]
\[ \exists x, y, z \text{ such that} \]
1. \( xyz = w \)
2. \( xy^k z \in L \) for \( k \geq 0 \)
3. \( |xy| \leq p \)
4. \( |y| > 0 \)

for all \( x, y \) exists.
3a) \[ L = \{ \text{ww} \mid w \in \{0,1\}^* \} \]

is not regular

Assume \( L \) is regular for the sake of contradiction.

Note \( 0^p 1^p \in L \) and \( 10^p 1^p \in L \),

thus

\[ \exists x y z \in x y z = w \]

\[ |y| > 0 \]

\[ |y| \leq p \]

\[ x y k z \in L \quad \forall k \geq 0 \]
Note: \( y = 0 \) if \( 1 > 0 \)

Thus \( xy^2z = 0^{p+i}1^p \)

Since \( i \neq 0 \)

\( p+i \neq p \)

Thus \( xy^2z \notin L \)

But by pumping lemma it should be

**Contradiction**

\( 3n \)
Note $0^3P^1 \leq L$ and $10^3P^1 \geq P$

$\exists \forall y \in L \ \exists \forall z \in L \ \forall k \geq 0$

$xyz = 0^3P^1$

$1xy1 \leq P$

$1y1 \geq 0$

$\forall \forall z \in L \ \forall k \geq 0$

$xy^2z = 0^3P^1 \ \forall$

Either $4P^1 \ i$ is not divisible by 4 (in which case not in)

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or $4p + i = 4j$

but $j > p$ since $i > 0$

thus $w$ must include 0s, but no 1s earlier, so contradiction

thus $xy^2z \notin L$

but by pumping (lemma it should be Contra diction)
3c. try
\[ w = \frac{OPAPOP}{WQWWWW} \]

3d. try
\[ w = \frac{OP10P102P}{W} \]
\[ v = 0^p 1^p 1^p 0^p 1^p 0^p \]

\[ y = 0^i, \quad i > 0 \]

\[ x y^2 z = 0^p 1^p 2^p 1^p 0^p 1^p 0^p \]

String is length \(6p+i\)

if \(i\) is not divisible by \(3\)

\(x y^2 z\) can't be in \(L/\)

thus \(6p+i = 0\) for \(i \geq 2p\)

So far \(x y^2 z = S^R S^S S^S\),

since \(|S^R| = j > 2p\) and \(p+i \leq 2p\),

\(S^R\) contains \(p+i\) Os,

but no contiguous chain of \(p+i\) Os elsewhere,

so contradiction
\[ w = 0^p 1 0^p 1 0^2 p \]

thus

\[ y = 0^i, \ i > 0 \]

therefore

\[ xy^2 z = 0^p t i 1 0^p 1 0^2 p \]

but \[ 2P+i > 2P \]

thus \[ xy^2 z \notin L \]

but by pumping lemma

it should be.

(contradiction)
Either show:

1. There exists a regular language L such that all subsets L' of L are regular.

Proving true, or

2. There exists a regular language L such that there is no subset L' of L such that L' is not regular.

Proving false

(it is false)
Example

$L = \{ 30, 18 \}^*$

$L' = \{ 3^n \mid n \geq 0 \}$

$L = \{ 13 \}^*$

$L' = \{ 3^n \mid n \text{ is prime} \}$