These are the doc cam sheets from the 4/16 class and the 4:50-6:05 pm review session, so that if you did not have time to copy them into your notes you can have them anyway. (However, these were hand-done on the spot, and so no promises are made about them; there could be typos or errors. And as 4/16 was done when we were running out of time, it at the top & bottom skips (saying “WHATEVER goes here”) things that are crucial parts of any answer. Also, on 4/16 & 4/17, the cleaning up I did on the board about removing the cardinality bars and adding a final “3” may not have made it onto the doc cam sheets, so this scanned version is better than the version I had at the review session. ) Cheers, Lane
\[ L_{LV/14} = \{ \langle M_1, M_2, M_3, x \rangle \mid x \in L(M_1) \cap L(M_2) \land x \notin L(M_3) \} \]

Suppose \( L_{LV/14} \) is decidable via decision \( R \).

We build a decision \( S \) for \( \text{Alt-M} \).

\( S \) on input \( \langle M, w \rangle \) does:

1. Build a TM \( \hat{M} \) that on arbitrarily ignores \( y \) and rejects.
2. Build a TM \( M' \) that on arbitrarily ignores \( y \).
3. Simulate \( R(\langle M, M, \hat{M}, w \rangle) \) and accept if \( R \) accepts, reject if \( R \) rejects.

(Note: \( R \) never loops as it is a decision)

End of \( S \)'s program.

If \( \langle M, w \rangle \in \text{Alt-M} \) then

\( \langle M, M, \hat{M}, w \rangle \in L_{LV/14} \) so \( R \) accepts \( \langle M, M, \hat{M}, w \rangle \) so \( S \) accepts \( \langle M, w \rangle \).

If \( \langle M, w \rangle \notin \text{Alt-M} \) then

\( \langle M, M, \hat{M}, w \rangle \notin L_{LV/14} \) as \( x \notin L(M) \),
so \( R(\langle M, M, M', w \rangle) \) so \( S \) rejects \( \langle M, w \rangle \),
so \( S \) is a decider for \( \text{Alt-M} \). Contradiction!

So \( L_{LV/14} \) is not decidable.
\[ L_{\text{TM}} = \{ \langle M, w \rangle \mid L(M) \text{ is decidable?} \} \]

Assume \( L_{\text{TM}} \) is decided by decider \( R \).

We give a decider \( S \) for \( \text{Atm} \).

\[ S \text{ on input } \langle M, w \rangle \text{ does:} \]

1. Build a TM \( M' \) that on an arb. input \( y \), ignores \( y \) and immersively simulates \( M \) on \( w \).

2. Run \( R(\langle M' \rangle) \) and if \( R \) accepts then reject and if \( R \) rejects then accept (\( R \) is a decider so it never runs forever).

3. \[ \text{END of } S. \]

Note that if \( \langle M, w \rangle \in \text{Atm} \) then \( L(M') = \{\varepsilon\} \) and if \( \langle M, w \rangle \notin \text{Atm} \) then \( L(M') = \emptyset \).

Note that if \( \exists \langle M, w \rangle \mid L(M) \text{ is decidable?} \) is

\[ = \exists \langle M, w \rangle \mid L(M) \text{ is decidable \& \& } \exists \cup \varepsilon, \]

and is infinite.

So if \( \langle M, w \rangle \in \text{Atm} \) then \( R(\langle M, w \rangle) \)

\( M' \) rejects since \( \varepsilon \cup \exists \langle M, w \rangle \mid L(M) \text{ is decidable?} \)

is not finite, so \( S \) accepts \( \langle M, w \rangle \).

And if \( \langle M, w \rangle \notin \text{Atm} \) then \( R(\langle M, w \rangle) \) accepts as

\( \emptyset \cup \exists \langle M, w \rangle \mid L(M) \text{ is decidable?} \) is \( \emptyset \) which is finite,

so \( S \) rejects \( \langle M, w \rangle \).

So \( S \) is a decider for \( \text{Atm} \), which is a contradiction, so \( L_{\text{TM}} \) is not decidable.
\[ L_{4/16.2} = \exists \langle M \rangle \mid L(M) \cap \exists \langle M' \rangle \mid L(M') \text{ is finite? is decidable?} \]

Assume \( L_{4/16.2} \) is decided by decider \( R \).

We give decider \( S \) for \( ATM \).

\( S \) on input \( \langle M, w \rangle \) does:

1. Build a TM \( M' \) that on all inputs \( y \) accepts. Immersively simulate \( M \) on \( w \).

2. Run \( R(\langle M' \rangle) \) and if \( R \) accepts then reject and if \( R \) rejects then accept.

\((R \) is a decider so never loops.\)

3. END of \( S \).

Note that if \( \langle M, w \rangle \in ATM \) then \( L(M') = \varepsilon \) and \( L(M) = \emptyset \).

If \( \langle M, w \rangle \in ATM \) then \( R(M') \) rejects since by separate proof we can show the TM \( M' \) is finite is undecidable, so \( S \) accepts.

If \( \langle M, w \rangle \notin ATM \) then \( R(M') \) accepts as \( \emptyset \cap \exists \langle M' \rangle \mid L(M') \text{ is finite?} = \emptyset \) which is decidable, so \( S \) rejects.

So \( S \) is a decider for \( ATM \), but \( ATM \) is undecidable so this is a contradiction, so \( L_{4/16.2} \) is not decidable.
\[ L_{4/6.3} = \exists \langle m_1, m_2, m_3 \rangle \ni L(m_1) \cap L(m_2) \subseteq L(m_3) \]

std. stuff goes here

\[ S \text{ on input } \langle m, n \rangle \text{ does:} \]

\[
\begin{align*}
\text{Build a TM } M \text{ that on arb. inputy is greedy and rejects.} \\
\text{Build a TM } M' \text{ that on arb. inputy immersively simulates } M \text{ on } w. \\
\text{Run } R(\langle M', M', M' \rangle) \text{ and if R accepts then reject if R rejects then accept (R is a decide if never loops?)}
\end{align*}
\]

END S

proof that S is a decision for \( A_{TM} \)

so we have contradiction goes here