

Department of Computer Science
CSC 247/447
Fall 2006

Assignment 5

Due: 5:00pm, Friday, December 8

Problem 1 (Model theory)

Consider the following model (for a first-order language with individual constants A, B, C and predicate constants P, Q, R):

$$\mathcal{M} = (\mathcal{D}, \mathcal{I}); \quad \mathcal{D} = \{i, j, k\};$$

$$\mathcal{I}(A) = i, \quad \mathcal{I}(B) = j;$$

$$\mathcal{I}(P) = \{i\}, \quad \mathcal{I}(Q) = \{i, j\}, \quad \mathcal{I}(R) = \{\langle i, j \rangle, \langle j, k \rangle\}.$$

For each of the following formulas, say whether it is true or false in \mathcal{M} , and justify your answer using a purely *semantic* argument (with no *syntactic manipulations*):

(a) $P(B)$, (b) $(\neg P(B) \Rightarrow Q(B))$, (the negation has narrow scope)

(c) $(\exists x R(x, x))$, (d) $(\forall x (Q(x) \vee R(B, x)))$,

(e) (Most x : $(x=x) Q(x)$)

Use the notation “ $\mathcal{M} \models \phi$ ” for “ ϕ is true in model \mathcal{M} ”. For (a,b), use the definition of truth in a model that applies to the variable-free case. For (c), use the definition of truth that requires the formula to be satisfied by every variable assignment. For (d) and (e), you need only give a sketchy justification of your answers – enough to make clear that you could fill in the details if asked to. For those who have not taken CSC 244/444 (and as a review for those who have), two semantic arguments of the sort required here are given in the Appendix (and also a “bad” argument involving syntactic manipulation).

(f) (*grads only – but undergrads can do it for extra credit*) Extend the interpretation function given above so that it also interprets the predicate operator “*very*” in such a way that $\mathcal{I}(\text{very})$ is a *subsetive, total* function on the power set of \mathcal{D} (and is not the identity function). Simply spell out the values of $\mathcal{I}(\text{very})$ on all possible arguments. Given this interpretation, what are the semantic values of $\mathcal{I}(\text{very})(P)$ and $\mathcal{I}(\text{very})(Q)$ in your extended model?

Problem 2 (Entailment and proofs)

(a) Which of the following inference rules are sound (i.e., the premises entail the conclusion) and which ones are unsound? For ones that are sound, briefly give intuitive reasons why the rule can only give a true conclusion from true premises, regardless of the model used. For ones that are unsound, specify a model $\mathcal{M} = (\mathcal{D}, \mathcal{I})^1$ such that there are premises that are true in \mathcal{M} (say what they are), from which the rule draws a conclusion that is false in \mathcal{M} (say what it is; you needn't prove the truth and falsity claims.)

(i) $\frac{\pi(\alpha) \vee \pi(\beta)}{\pi(\gamma)}$, where π is any predicate and α, β, γ are any constants.

(ii) $\frac{\pi(\alpha)}{(\pi(\alpha) \vee \pi(\beta))}$, where π is any predicate and α, β are any constants.

(iii) $\frac{(\exists x \pi(x))}{\pi(\alpha)}$ where π is any predicate and α is any constant;

(iv) $\frac{(\forall x \pi(x))}{(\exists x \pi(x))}$.

(b) Though the rule $\frac{\text{Bird}(\tau)}{\text{Has-beak}(\tau)}$ is not sound, it will always yield true conclusions in models wherein a certain general premise is true. What is that premise (expressed as a first-order logical formula)? Informally state why the conclusions obtained are true in the models at issue.

(c) In the quick review of deduction in class, the rules of *modus ponens* (MP) and universal instantiation (UI) were mentioned and illustrated. Two additional useful rules are existential instantiation (EI, also called Skolemization) and AND-elimination (AE):

$$\text{EI: } \frac{(\exists x \phi)}{\phi_{Sk/x}}$$

$$\text{AE: } \frac{(\phi \wedge \psi)}{\phi}, \frac{(\phi \wedge \psi)}{\psi}$$

where the subscript Sk/x in EI indicates substitution of a new constant Sk for all free occurrences of x in ϕ . The converse of AE, AND-introduction (AI), will also be needed. Using the five rules mentioned, prove that

$$\text{NYC} = \text{BigApple},$$

given that

1. $\exists e. \text{Born}(e, \text{Salinger}, \text{NYC});$

2. $\exists e. \text{Born}(e, \text{Salinger}, \text{BigApple});$

3. $\forall x. \forall e. \forall y. \forall e'. \forall y'. [\text{Born}(e, x, y) \wedge \text{Born}(e', x, y')] \Rightarrow (e' = e \wedge y' = y).$

Here we're using the "dot" convention to avoid writing many brackets, where a dot following a quantifier and variable indicates that the scope of that quantifier goes all the way to the right end of the formula (unless and until an unbalanced right bracket is encountered).

¹with \mathcal{D} nonempty, as usual

Problem 3 (Scope preferences and events)

Consider the sentence

All hikers with a pair of binoculars said that they saw a sasquatch.

- (a) Show the initial unscoped LF of this sentence (without introducing event variables, and treating past tense as an unscoped operator), using a relational semantics in your interpretation of “*said*”, “*with*” and “*saw*”. Ignore the plural in *hikers*. Also ignore all lexical ambiguities (such as ambiguities in the meaning of *with* and *see*). Treat *pair of binoculars* as a single lexical entry, say with LF **pair-bin**. Assume that *say* (and thus *said*) is a predicate taking two arguments (neglecting the implied event for now), namely a subject (who does the saying) and a propositional object. The propositional object is formed by applying a “reifying” operator, **that**, to the subordinate clause.² At this point, leave *they* as simply **they**, instead of trying to give it a more specific interpretation.

Also show the LFs of the \bar{N} “*hikers with a pair of binoculars*”, and of the VP “*saw a sasquatch*”, as they appear prior to incorporation into the complete sentential LF. Also show the phrase structure rule and LF rule for the \bar{N} and the VP, that you used to obtain your LFs. You don’t need to draw a PST or show LFs for all lexical items or phrases (but perhaps you’ll find it helpful to do so, in obtaining the asked-for results).

- (b) Introduce ‘classical’ Davidsonian event variables into the LF in (a), treating them as λ -abstracted variables that we “carry upward” to the sentence level re-binding and re-abstracting them as needed. Assume that as long as the **past** operator remains unscoped, the LF of a sentence still has a top-level lambda-abstracted event variable. (When **past** is scoped, the event variable becomes existentially quantified and is predicated to be in the past.)

Much as in (a) also show the LF of the VP “*saw a sasquatch*”, as it would appear in a bottom-up LF computation, and show the modified semantic rule you used to obtain this LF.

- (c) Show the unscoped LF of the subordinate clause only, for a “neo-Davidsonian” approach to the verb LF, with *verb-specific* thematic roles (make the best choices of roles that you can, using the possibilities on p. 250 of the Allen text). Also show the appropriate lexical entry for the verb, and the LF of the subordinate VP, again carrying along the event variable as a λ -abstracted variable.
- (d) Show a possible final scoping of the LF in (a), choosing one that you regard as corresponding to a “natural” reading of the sentence. (At this point, replace **they** by an appropriate bound variable.) Then discuss this scoping in relation to the scoping preferences that were covered in class, explaining to what extent the reading you have chosen is consistent with those preferences, and where they are not, what the reasons (in terms of world knowledge, or familiar patterns of usage, etc.)

²A reifying operator maps entities other than individuals – in this case, sentence meanings – into the domain of individuals, \mathcal{D} .

might be. Also show one other possible scoping, and explain the intuitive meaning of this scoped LF.

- (e) Show what the first LF you gave in (d) would become if you used Davidsonian event variables, as in (b).

Problem 4 (Discourse and understanding)

- (a) What lexical ambiguities are present in the sentence in problem (3), and what techniques might be used to resolve them? (Discuss only those lexical ambiguities that are relevant to the “natural” parse of the sentence, as assumed in (3); e.g., do not consider the noun sense of *saw*.)
- (b) Which syntactic constraint(s) on interpreting anaphora, discussed in class, come(s) into play in the interpretation of *they* in problem (3), and how?
- (c) What possible intrasentential antecedents can the two pronouns in the following sentence have? Explain what constraints you are applying.

He knows that Bill doesn't share John's opinion of him.

- (d) Give examples of several entities that are made available for subsequent reference by the sentence

John ran into Mary at the zoo,

even though there are no explicit phrases denoting those entities in the sentence. To illustrate their availability, use immediately following sentences referring to them.

- (e) Explain the coherence of

John was hungry. He resisted going to the vending machine.

by considering in step-by-step manner the inferences (or hypotheses) suggested by each of the sentences, and the way they can be linked.

Appendix (Sample model theoretic proofs)

A semantic proof is one that refers only to the (recursive) definition of *truth* (and in the variable case, *satisfaction*) relative to a model (and variable assignment). Syntactic manipulations like $\neg(P(A) \wedge \neg P(A)) \mapsto (\neg P(A) \vee P(A))$ are not allowed, since these ultimately require semantic justifications themselves, showing that the original formula is true in a model iff the transformed formula is true in it!

The following are two semantic arguments for the truth of certain formulas in a model $\mathcal{M} = (\mathcal{D}, \mathcal{I})$ where $\mathcal{D} = \{d, e, f, g\}$, $\mathcal{I}(A) = d$, and $\mathcal{I}(P) = \{d, e, f\}$. The first uses the definition of truth suitable for variable-free formulas, and the second illustrates the more general definition of truth and satisfaction for the case where variables are present.

Concerning the notation “ $\mathcal{U}_{x:\delta}$ ” in the second proof, this stands for “the variable assignment \mathcal{V} which is the same as \mathcal{U} except that $\mathcal{V}(x) = \delta$. So to say that for all $\delta \in \mathcal{D}$, ϕ is satisfied by $\mathcal{U}_{x:\delta}$ relative to model \mathcal{M} is the same as saying that for all $\delta \in \mathcal{D}$, ϕ is satisfied by \mathcal{V} where \mathcal{V} is the same as \mathcal{U} except that $\mathcal{V}(x) = \delta$.

1. $\mathcal{M} \models \neg(P(A) \wedge \neg P(A))$
iff not $\mathcal{M} \models (P(A) \wedge \neg P(A))$ (by the truth conditions for \neg);
iff not [$\mathcal{M} \models P(A)$ and $\mathcal{M} \models \neg P(A)$] (by the truth cond’s for \wedge);
iff not [$\mathcal{I}(A) \in \mathcal{I}(P)$ and not $\mathcal{M} \models P(A)$] (by the truth cond’s for $P(A)$ and \neg);
iff not [$d \in \{d, e, f\}$ and not $d \in \{d, e, f\}$] (by the def’n of \mathcal{I});
iff not $d \in \{d, e, f\}$ or $d \in \{d, e, f\}$;

but this is clearly true since the second disjunct is true. (*Note:* we could easily have generalized this argument so that it applies to *any* model; the last statement would have been “either not $\mathcal{I}(A) \in \mathcal{I}(P)$, or $\mathcal{I}(A) \in \mathcal{I}(P)$, which is true regardless of \mathcal{I} . So this is a *valid* formula, i.e., one that is true in *all* models.)

In view of the above remarks about syntactic manipulation, it would have been inadmissible to argue in the following sort of way:

$\mathcal{M} \models \neg(P(A) \wedge \neg P(A))$ iff $\mathcal{M} \models (\neg P(A) \vee P(A))$ iff $\mathcal{M} \models \neg P(A)$ or $\mathcal{M} \models P(A)$, etc.

2. $\mathcal{M} \models \neg(\forall x \neg P(x))$
iff for all v.a. \mathcal{U} , $\mathcal{M} \models \neg(\forall x \neg P(x))[\mathcal{U}]$ (by the general def’n of truth in a model);
iff for all v.a. \mathcal{U} , not $\mathcal{M} \models (\forall x \neg P(x))[\mathcal{U}]$ (by the sat’n cond’s for \neg);
iff for all v.a. \mathcal{U} , not for all $\delta \in \mathcal{D}$, $\mathcal{M} \models \neg P(x)[\mathcal{U}_{x:\delta}]$ (by the sat’s cond’s for \forall);
iff for all v.a. \mathcal{U} , not for all $\delta \in \mathcal{D}$, not $\mathcal{M} \models P(x)[\mathcal{U}_{x:\delta}]$ (by the sat’s cond’s for \neg);
iff for all v.a. \mathcal{U} , for some $\delta \in \mathcal{D}$, $\mathcal{M} \models P(x)[\mathcal{U}_{x:\delta}]$;
iff for all v.a. \mathcal{U} , for some $\delta \in \mathcal{D}$, $\delta \in \mathcal{I}(P)$;

but the last statement is clearly true, as shown by the choice $\delta = d$, since $d \in \mathcal{I}(P) = \{d, e, f\}$, regardless of the v.a. \mathcal{U} .